

Seet 2.1.

Pr 23 Let  $x$  be a solution of

$$(1) \quad Ax = 0$$

Multiply to the left by  $C$

$$\underbrace{(CA)}_{I_m} x = C \cdot 0 = 0$$

So

$$x = 0$$

We just found out that the only solution of (1) is  $x = 0$ .

Assume  $A$  is  $m \times n$  with  $m < n$ . Then  $A$  has at most  $m$  pivot positions, hence there are non-pivot columns, hence (1) has more than one solution. Contradiction!  
So  $m \geq n$ .

Pr 24  $(0b)$  is always a solution of

$$(2) \quad Ax = b$$

because  $A(0b) = \underbrace{(AD)}_{I_m} b = I_m b = b$

If  $m > n$  ( $A$  has more rows than columns)  
then  $A$  has at most  $n$  pivot positions so  
there are rows with no pivot positions  
so (2) becomes inconsistent for certain choices  
of  $b$  (see Theorem 4 in Sect 1.4). Contradiction!  
Thus  $m \leq n$ .

Pr 25  $CA = I_m \Rightarrow m \geq n$  by problem 23  
 $AD = I_m \Rightarrow m \leq n$  by problem 24  
So  $m = n$ .

$$CA D = C(AD) = C I_m = C$$

On the other hand

$$CA D = (CA) D = I_n D = D$$

Hence  $C = CA D = D \Rightarrow C = D$ .