Math 225, Section M1

Review Sheet for Final Exam

May 3, 2009

1. **Linear System of Equations.** Know how to:

   - write them in vectorial and matrix form, see Sections 1.3 and 1.4;
   - solve them using:
     - row reduction, see Section 1.2;
     - inverse of the matrix of the system (works only when the number of the unknowns equals the number of equations), see Section 2.2;
     - Cramer rule (works only when the number of the unknowns equals the number of equations), see Section 3.3;
   - write their solutions in (parametric) vector form, see Section 1.5.

   You should also know when the linear system of equations (in matrix form) $Ax = b$:

   - has no solution (inconsistent), or, on the contrary, has at least one solution (consistent) and how are these situations related to the span of the columns of A, or to Col$A$, see Sections 1.2, 1.4 and 4.2;
   - has at most one solution, or, on the contrary, infinitely many solutions and how are these situations related to: the linear independence/dependence of the columns of A, or Nul$A$, or the dimension Nul$A$, see Sections 1.7, 4.2, 4.5.

2. **Matrix Operations and Invertible Matrices.** Know:

   - how to add two matrices, how to multiply a matrix by a number (scalar), how to multiply two matrices, how to take the transpose of a matrix, and the properties of all these operations, see Section 2.1;
   - the definition of an invertible matrix, see Section 2.2;
   - how to use invertible matrices to solve systems of equations, see Theorem 5 in Section 2.2;
   - how to compute the inverse of a matrix:
     - for a 2×2 matrix, see Theorem 4 in Section 2.2;
     - via row operations (row reduction), see Section 2.2;
     - using determinants, see Section 3.3;
   - properties of the inverse of a matrix, see Theorem 6 in Section 2.2;
   - how to characterize the invertibility of a square matrix A via:
     - linear independence of columns of A, or the number of solutions of $Ax = 0$, or the Nul$A$, or the dimension of Nul$A$, see Theorem 8 in Section 2.3 and Theorem in Section 4.6;
     - consistency of $Ax = b$, or the span of columns of A, or ColA, or the dimension of ColA, or rank$A$, see Theorem 8 in Section 2.3 and Theorem in Section 4.6;
     - the determinant of A, see Theorem 4 in Section 3.2.
     - the invertibility of the transpose of A, see Theorem 8 in Section 2.3.

3. **Determinants.** Know:

   - the definition of the determinant of a 1×1 matrix, 2×2 matrix, larger matrices via cofactor expansions, see Sections 3.1;
   - how a determinant changes after a row operation or when taking the transpose, or after multiplying two matrices, see Section 3.2;
   - how to calculate determinants using row, column operations and cofactor expansions, in particular how to compute determinants of upper or lower triangular matrices;

- how to use determinants to:
  - establish whether a matrix is invertible, see Theorem 4 in Section 3.2;
  - solve linear system of equations using the Cramer rule, see Section 3.3;
  - calculate the inverse of a matrix, see Section 3.3;
  - calculate the area of a parallelograms or volumes of parallelepipeds, see Section 3.3;

4. **Vector Spaces.** Know:

   - the definition of a vector space, properties of the operations in a vector space, definition of span, see Section 4.1;
   - the definition of a subspace, how to check whether given sets are subspaces in particular why for an $m \times n$ matrix A, Nul$A$ and Col$A$ are subspaces of $\mathbb{R}^n$ respectively $\mathbb{R}^m$, see Sections 4.1 and 4.2;
   - the definition of linear dependence and linear independence of a set vectors and how to check whether a given set of vectors is linearly dependent/independent, in particular the relation between linear dependence and independence of vectors in $\mathbb{R}^n$ and the number of solutions of the equation $Ax = 0$, see Sections 4.3 and 1.7;
   - the definition of a basis in a vector space, the definition of the dimension of a vector space, how to find basis and the dimension of Nul$A$ and Col$A$ where A is an $m \times n$ matrix, see Section 4.3 and 4.5;
   - the definition of the rank of a matrix, the Rank Theorem and its applications to systems of linear equations and to establishing the invertibility of a matrix, see Section 4.6;

5. **Eigenvalues, Eigenvectors and Diagonalization of Square Matrices** Know:

   - the definition of an eigenvalue and an eigenvector, see Section 5.1;
   - how to compute the eigenvalues and the corresponding eigenspaces for given square matrix, see section 5.2;
   - when is a square matrix diagonalizable and how its eigenvalues and eigenvectors are used to diagonalize a matrix, see section 5.3;
   - how to use diagonalization in computing the powers of a square matrix, see Section 5.3.

6. **Inner Product, Distance, Orthogonality and Least Square Problems** Know:

   - the definition of inner product of vectors in $\mathbb{R}^n$ and its properties, the definitions of the length of a vector and distance between two vectors, see Section 6.1;
   - the definition of orthogonality and Pytagora's Theorem, see Section 6.1;
   - the definition of orthogonal sets of vectors in $\mathbb{R}^n$ and their linear independence, see Section 6.2;
   - the definition and how to calculate the orthogonal projection of a vector onto a subspace of $\mathbb{R}^n$, see Section 6.3;
   - the definition of least squares solutions and how to calculate them, see Theorem 13 in Section 6.5.