1) Let $D$ be a domain in $\mathbb{C}$ and $(f_n)$ be a sequence of uniformly bounded analytic functions on $D$. Let $S$ be a subset of $D$ such that $S$ has a limit point in $D$, and $(f_n)$ converges pointwise on $S$. Prove that $(f_n)$ is uniformly Cauchy on compact subsets of $D$.

2) Let $\mathfrak{F}$ be a set of analytic functions on $D$. Show that $\mathfrak{F}$ is relatively compact in $A(D)$ if and only if $\exists M_n \geq 0$ with $\limsup_{n} \sqrt{M_n} \leq 1$ and $\forall f \in \mathfrak{F}, \forall N \geq 0$, \[
\left| \frac{f^{(n)}(0)}{n!} \right| \leq M_n.
\]

3) Construct explicitly a meromorphic function on $\mathbb{C}$ with poles exactly at $in, n = 1, 2, 3, \ldots$, and corresponding principal part \[
\frac{n^{5/2}}{(z-in)^2}.
\]

4) Let $D = \{ z \in \mathbb{C} : 0 < \text{Im } z < \pi \}$ and let $\mathfrak{F}$ denote the set of conformal mappings $f : D \to \mathbb{D}$ with $f \left( \frac{i\pi}{2} \right) = 0$.
   (i) Construct explicitly a function $h \in \mathfrak{F}$.
   (ii) Determine the set of all possible values of $f' \left( \frac{i\pi}{2} \right)$ for $f \in \mathfrak{F}$ and justify your answer.

5) Let $G = \left\{ z \in \mathbb{C} : \frac{\pi}{4} < \text{Arg} \frac{1+z}{1-z} < \frac{\pi}{2} \right\}$.
   (i) Find the equation of the boundary of $G$ in $x, y$ coordinates.
   (ii) Find a conformal mapping of $G$ onto the open disk $\mathbb{D}$.

6) Suppose that $F$ is analytic on a convex domain $\Omega$.
   (i) Suppose that $F'(z) \neq 0$ for all $z \in \Omega$. Give an example to show that $F$ need not be one-to-one.
   (ii) Suppose, in addition, that $\text{Re } F'(z) > 0$ for all $z \in \Omega$. Prove that $F$ is one-to-one.

7) Show that every conformal map of the upper half plane $\mathbb{H} = \{ z \in \mathbb{C} : \text{Im } z > 0 \}$ is of the form \[
z \mapsto \frac{az + b}{cz + d}, \quad a, b, c, d \in \mathbb{R}, \quad ad - bc = 1.
\]

8) (i) Find explicitly a conformal mapping of the horizontal strip $\{ z \in \mathbb{C} : 0 < \text{Im } z < 1 \}$ onto $\mathbb{D}$. 


(ii) Find a conformal mapping of \( \{ z \in \mathbb{D} : \text{Im} \, z > 0 \} \) onto \( \mathbb{D} \).

9) (i) Find explicitly a conformal mapping from \( \mathbb{D} \setminus [0, 1) \) onto \( \mathbb{D} \).
   (ii) Find an analytic mapping from \( \mathbb{D} \setminus [1/2, 1) \) onto \( \mathbb{D} \).

10) (i) Find a conformal mapping \( f \) from
    \[
    \Omega = \{ z \in \mathbb{C} : |z + i| < \sqrt{2}, |z - i| < \sqrt{2} \}
    \]
    onto \( \mathbb{D} \) such that \( f(0) = 0 \).
    (ii) Is there a conformal mapping \( g : \Omega \to \mathbb{D} \) such that \( g(0) = 0 \) and \( g(1/2) = 9/10 \)? Justify your answer.