Math 347 Exam #1 Spring 2005

Name _____

All problems except # 1 require proofs and/or explanations.

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1. (16 points) Write the negation of each statement.

   (a) $\forall x \in \mathbb{R}$, if $x^2 > x$, then $x < 0$ or $x > 1$.

   Negation:

   (b) $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$ such that for each integer $n \geq N$, $|a_n - L| < \varepsilon$.

   Negation:

2. (12 points) Let $S = \{x \in \mathbb{R} : |x + 1| > x\}$. Prove that $S = \mathbb{R}$.

3. (24 points) (a) Let $n \in \mathbb{Z}$. Prove that n is even if and only if $3n - 2$ is even.

   (b) Show that if $f : \mathbb{R} \to \mathbb{R}$ is increasing, then $g(x) = -f(-x)$ is also increasing.

4. (16 points) Prove that for all $n \in \mathbb{N}$, $\displaystyle\sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3}$.

5. (22 points) For each statement, determine whether it is true or false and give a proof.

   (a) For all functions $f : \mathbb{R} \to \mathbb{R}$, if f is bounded, then $h(x) = (f(x))^2$ is bounded.

   (b) For all sets A and B, if $A - B = \emptyset$, then $A = B$.

6. (10 points) Suppose $P(1), P(2), P(3), \ldots$ is an infinite sequence of statements. Also, suppose that for all $n \in \mathbb{N}$, if $P(n)$ and $P(n+1)$, then $P(n+5)$ (think of this as an inductive step). What statements need to be proved in a base step in order to conclude that $P(n)$ is true for all n ? **Explain your reasoning.**