

Math 453 – Number Theory
Exam 1 October 12, 2005

Make sure to show all of your work on # 1, 2 and 5. Say what theorems you use to derive your answers.

1. (24 points)
 - (a) Find $[45, 54]$.
 - (b) Determine how many integers between 1 and 1500 have an inverse modulo 1500.
 - (c) Find $(930012023786827, 1000000000000)$. Do not use the Euclidean algorithm.
2. (20 points)
 - (a) For what values of c does the equation $18x - 87y = c$ have solutions in integers x, y ?
 - (b) Find all solutions of the congruence $x^4 \equiv 4x^2 \pmod{p}$, where p is a prime, $p \geq 3$.
3. (13 points) Prove that if $3|a$ and $4|b$, then $12|(8a - 9b)$.
4. (18 points) Prove that if p is a prime and $p \geq 5$, then $p^2 \equiv 1 \pmod{12}$. Hint: determine what remainders are possible when p is divided by 12.
5. (25 points)
 - (a) Find the remainder when 5^{80} is divided by 7, and the remainder when 5^{80} is divided by 41.
 - (b) Use part (a) to find the remainder when 5^{80} is divided by 287 (note that $287 = 7 \cdot 41$).