

Math 453 – Number Theory
Exam 2 December 2, 2005

Make sure to show your work and state what theorems you use.

1. (18 points)
 - (a) Find the number of positive divisors of 600.
 - (b) Find the sum of the positive divisors of 600.
 - (c) Find the largest order of any number modulo 600.
2. (16 points)
 - (a) State a definition of $\text{ord}_n x$.
 - (b) Suppose that $x^2 \equiv -1 \pmod{n}$. Determine $\text{ord}_n x$ (give a proof).
3. (34 points)
 - (a) (9 points) Determine how many primitive roots each number has.
 - (i) 18
 - (ii) 19
 - (iii) 20
 - (b) (15 points) Prove that 2 is a primitive root of 5^k for *every* positive integer k .
3. (c) (10 points) Determine the number of solutions of each congruence.
 - (i) $x^6 \equiv 2 \pmod{5^3}$.
 - (ii) $x^7 \equiv 2 \pmod{5^3}$.
4. (12 points) Prove that the function $f(n) = n\tau(n^2)$ is a multiplicative function.
5. (20 points) Suppose that $\text{ord}_n b = k$. Prove that if k is even, then $\text{ord}_n b^2 = k/2$ and if k is odd, then $\text{ord}_n b^2 = k$.