Graduate Course Description

Fall 2013

Math 595: Additive number theory

Instructor: Kevin Ford

Time/place: MWF 1:00–1:50 (147 Altgeld)

Prerequisites: Math 531/equivalent or consent of the instructor.

Recommended Text: There is no official text for the course, but the following books contain much of the material.

Course Description. Roughly speaking, additive number theory is concerned with problems about sumsets

$$A_1 + A_2 + \cdots + A_k := \{a_1 + a_2 + \cdots + a_k : a_1 \in A_1, a_2 \in A_2, \ldots, a_k \in A_k\},$$

where $A_i$ are sets of non-negative integers. Of particular interest are problems where the sets $A_i$ are defined by multiplicative constraints, e.g. squares, cubes, $k$-th powers, primes, powers of 2, etc. If $A_1 = A_2 = \cdots = A_k = A$ and $A_1 + \cdots + A_k$ contains all non-negative integers $n$, we call the set $A$ a basis of order $k$. Fundamental problems include determining whether a given set is a basis or not (of some finite order), constraining the size or structure of $A + B$ given information about $A$ and $B$, or showing that sets with certain additive properties exist.

We will explore a variety of methods to attack additive number theoretic problems, starting with combinatorial methods, sometimes referred to as elementary methods (although by no means easy or trivial!), such as the Schnirelman theory of set addition, culminating in the proof that the primes form a basis. We will also discuss the probabilistic method and applications to the existence of “thin” bases. We will also explore the use of harmonic analysis to solve additive problems, such as proving Vinogradov’s theorem that all large odd integers are the sum of 3 primes, and solving Waring’s problem, that for every $k \geq 2$, the set of non-negative $k$-th powers forms a basis.

We will discuss a wide variety of other topics including Romanoff’s theorem, the Erdős-Fuchs theorem, Freiman’s theorem, sum-product theorems, and additive problems over finite fields.