

Math 347 Practice Exam #2

Name _____

All problems except # 2 require proofs and/or explanations.

- Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(n) = 3n + 1$.
 - Is f an injection ? _____ **Explain:**
 - Is f a surjection ? _____ **Explain:**
- Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$. Answer each as True or False (no explanation required).
 - _____ (a) if f is increasing, then f is surjective.
 - _____ (b) if f is surjective, then f is unbounded.
 - _____ (c) if f and g are bijections, then $g^{-1} \circ f^{-1}$ is injective.
- Define a relation on the set \mathbb{R} by $x \sim y$ if $|x - y| \leq 1$. Is \sim an equivalence relation?
- Suppose $n \in \mathbb{N}$ and $n \equiv 5 \pmod{6}$. Prove that $n^2 + 2$ is composite.
- Show that if a is even and b is odd, then $\gcd(a, b) = \gcd(a/2, b)$.
- Suppose that $a \equiv b \pmod{m}$ and that $k|m$. Prove that $a \equiv b \pmod{k}$.
- Suppose $\gcd(a, 21) = 1$. Use Fermat's Theorem to prove that $a^6 \equiv 1 \pmod{21}$.
- A drawer contains 7 red socks, 10 white socks and 8 blue socks. You grab three of the socks at random. What is the probability that all three are the same color?
- Let S be a square of side length 1. Let P_1, \dots, P_{101} be 101 points inside of S . Prove that two of the points are a distance $\leq \sqrt{2}/10$ apart. Hint: partition S into little squares of side $\frac{1}{10}$, and use the pigeonhole principle.