Math 353 – Number Theory

Exam 1   October 14, 2002

1. (20 points)

   (a) Give an example of a **reduced** residue system modulo 30.

   (b) Solve the congruence $40x \equiv 26 \pmod{14}$.

2. (15 points) Suppose $(a, b) = 1$. What are the possibilities for $(2a, 3b)$?

   Explain:

3. (20 points) (a) If $x \in \mathbb{Z}$, what are the possibilities (from 0 to 6) for x^3 modulo 7?

   (b) Let $n \equiv 3 \pmod{7}$. Show that the equation $x^3 + y^3 = n$ has no solutions in integers x, y.

4. (15 points) Prove that if p is a prime and $p + 1$ is a square, then $p = 3$.

5. (15 points) Suppose p and q are distinct primes, and let $L = [p - 1, q - 1]$. Prove that if $(a, pq) = 1$ then $a^L \equiv 1 \pmod{pq}$. (This does not follow from Euler's Theorem).

6. (15 points) Suppose $p_1, \ldots, p_k$ are distinct **odd** primes and let $m = p_1 \cdots p_k$. Show that the congruence $x^3 \equiv x \pmod{m}$ has exactly 3^k solutions x modulo m.