Problem 3. Let $n \geq 0$ and let $X$ be a space with the homotopy type of a CW complex. Consider the Postnikov truncation map $t_n : X \to P_n X$, which may be assumed a relative CW complex.

Let $f : X \to Z$ be any map, where $Z$ is an Eilenberg-MacLane space of type $(G, k)$ for some abelian group $G$ and $k \leq n$.

Show that there exists a map $g : P_n X \to Z$ satisfying $f \simeq g \circ t_n$, and this map $g$ is unique up to homotopy. Here, $g$ makes the diagram

\[
\begin{array}{ccc}
X & \xrightarrow{t_n} & P_n X \\
\downarrow f & & \downarrow g \\
& Z & \\
\end{array}
\]

commute up to homotopy.

Remark. The statement still holds when $Z$ is a product of such Eilenberg-MacLane spaces. However, if $Z$ is more complicated, but still $n$-truncated (i.e. $\pi_i(Z) = 0$ for $i > n$), then such a factorization $g : P_n X \to Z$ still exists, but its homotopy class need not be unique.