Problem 1. (May § 10.7 Problem 2)

a. Let \( f : X \xrightarrow{\sim} Y \) be a weak homotopy equivalence. Assuming \( X \) is a CW-complex and \( Y \) has the homotopy type of a CW-complex, show that \( f \) is a homotopy equivalence.

b. Show that the space \( A := \{0\} \cup \{\frac{1}{n} \mid n \in \mathbb{N}\} \subset \mathbb{R} \) does not have the homotopy type of a CW-complex.

Problem 2. Consider the “equatorial” embeddings

\[ S^0 \subset S^1 \subset S^2 \subset \ldots \]

of spheres, and define the infinite-dimensional sphere \( S^\infty := \text{colim}_n S^n \). Show that \( S^\infty \) is contractible.