Problem 1. Show that $\int_1^\infty \frac{1}{x^p} dx$ converges if $p > 1$.

sol) Note that $\lim_{b\to\infty} \frac{1}{b^{p-1}} = 0$ if $p > 1$, since $p - 1 > 0$.
And $\lim_{b\to\infty} \frac{1}{b^{p-1}} = \lim_{b\to\infty} b^{1-p} = \infty$ if $p < 1$, since $1 - p > 0$.
Hence, if $p > 1$, then we have

$$\begin{aligned}\int_1^\infty \frac{1}{x^p} dx &= \lim_{b\to\infty} \int_1^b x^{-p} dx \\ &= \lim_{b\to\infty} \frac{1}{1-p} \frac{1}{x^{p-1}} \Big|_1^b \\ &= \lim_{b\to\infty} \frac{1}{1-p} \frac{1}{b^{p-1}} - \frac{1}{1-p} = \frac{1}{p-1}.\end{aligned}$$

Problem 2. Show that $\int_0^1 \frac{1}{x^p} dx$ converges if $p < 1$.

sol) Note that $\lim_{a\to 0} \frac{1}{a^{p-1}} = \infty$ if $p > 1$, since $p - 1 > 0$.
And $\lim_{a\to 0} \frac{1}{a^{p-1}} = \lim_{a\to 0} a^{1-p} = 0$ if $p < 1$, since $1 - p > 0$.
Thus, if $p < 1$, we get

$$\begin{aligned}\int_0^1 \frac{1}{x^p} dx &= \lim_{a\to 0} \int_a^1 x^{-p} dx \\ &= \lim_{a\to 0} \frac{1}{1-p} \frac{1}{x^{p-1}} \Big|_a^1 \\ &= \lim_{a\to 0} \frac{1}{1-p} - \frac{1}{1-p} \frac{1}{b^{p-1}} = \frac{1}{1-p}.\end{aligned}$$