Problem 1 Determine whether each sequence $\{a_n\}$ converges of diverges. If it converges, find the limit.

(a) $a_n = \frac{2n+3}{n-1}$

(b) $a_n = (-1)^n$

(c) $a_n = \frac{n}{\ln n}$

(d) $a_n = \frac{10 + \sin n}{n}$

Problem 2 Determine whether the given series converges or diverges.

(a) $\sum_{k=1}^{\infty} \frac{1}{k^{1.5}}$

(b) $\sum_{k=1}^{\infty} \frac{1}{k^{0.5}}$

Problem 3 Find the sum of the series.

(a) $\sum_{k=1}^{\infty} \frac{3^{k+1}}{5^k}$

(b) $\sum_{k=1}^{\infty} \frac{1}{k(k+2)}$

Problem 4 (a) State the k-th term divergence test.

Hypothesis:

Conclusion:

(b) Determine whether $\sum_{k=1}^{\infty} 2^{-\frac{\ln k}{k}}$ converges or diverges.

Problem 5 (a) State the integral test.
Hypothesis:

Conclusion:

(b) Determine the series $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ is convergent or divergent.

Problem 6 (a) State the Comparison test.

Hypothesis:

Conclusion:

(b) Determine whether the given series converges or diverges.

i) $\sum_{k=2}^{\infty} \frac{k^2}{k^3 - 3}$

ii) $\sum_{k=1}^{\infty} \frac{1}{k + 2^k}$

Problem 7 (a) State the Limit Comparison test.

Hypothesis:

Conclusion:

(b) Determine whether the given series converges or diverges.

i) $\sum_{k=1}^{\infty} \frac{\sqrt{k} + 1}{4k + 3}$

ii) $\sum_{k=1}^{\infty} \frac{k^2 + 4}{k^5 + 3k + 1}$

Problem 8 (a) State the Alternating series test for an alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$ with $a_k > 0$.

Hypothesis:

Conclusion:

(b) Determine whether the series converges or diverges.

i) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$

ii) $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{3^k}$

iii) $\sum_{k=6}^{\infty}(-1)^{k+1}e^{1/k}$

Problem 9 (a) State the Ratio test.

(b) Determine whether the series is convergent or divergent.

i) $\sum_{k=1}^{\infty} \frac{k^2}{10^k}$

ii) $\sum_{k=1}^{\infty} \frac{(2k+1)!}{3^k}$

iii) $\sum_{k=1}^{\infty} (-1)^k \frac{4^k}{k!}$

iv) $\sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k}$

Problem 10 (a) State the Root test.

(b) Determine whether the series is convergent or divergent.

i) $\sum_{k=1}^{\infty}\left(\frac{2k}{3k+1}\right)^k$

ii) $\sum_{k=1}^{\infty}\left(\frac{2+\ln k}{k}\right)^k$

Problem 11 Determine whether the series converges absolutely, converges conditionally or diverges.

(a) $\sum_{k=1}^{\infty} \frac{\cos k\pi}{\sqrt{k}}$

(b) $\sum_{k=1}^{\infty} \frac{\sin k}{k^{3/2}}$

(c) $\sum_{k=1}^{\infty} (-1)^k \frac{k+1}{k^2}$

Problem 12 Suppose that $\sum_{k=1}^{\infty} a_k$ is an infinite series with the property that $\sum_{k=1}^{n} a_k = \frac{\ln n}{n}$ for all positive integers n.
Does $\sum_{k=1}^{\infty} a_k$ converge? Justify your answer.

Problem 13 Show that if $a_k > 0$ for all k and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty} a_k^2$ converges.