Problem 1 Find the radius of convergence and the interval of convergence of each power series.

(a) $\sum_{k=0}^{\infty}(x-3)^k$

(b) $\sum_{k=0}^{\infty}(3x+1)^k$

(c) $\sum_{k=0}^{\infty}\frac{k!}{(2k)!}x^k$

(d) $\sum_{k=1}^{\infty} \frac{(-1)^k}{4k^2 - 1}(2x)^k$

(e) $\sum_{k=1}^{\infty} \frac{(x+4)^k}{k2^k}$

Problem 2 Find the power series of the form $\sum_{k=0}^{\infty} a_k x^k$ converging to each function. In each case, specify the radius of convergence of the power series.

(a) $\frac{2}{2-x}$

(b) $\frac{3}{(x-1)^2}$

(c) $\ln(1+x)$

(d) $\tan^{-1}x$

Problem 3 Find the Taylor series for e^{3x} in $x + 1$.

Problem 4 Find the Taylor series for $\sin x$ in $x - \pi$.

Problem 5 Evaluate each limit.

(a) $\lim_{x\to 0} \dfrac{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} - \cos x}{x^7 \sin x}$

(b) $\lim_{x\to 0} \dfrac{\tan^{-1} x - x + \frac{x^3}{3}}{x^5 \cos x}$

Problem 6 Use the first three nonzero terms of a Taylor series to approximate $\cos\left(\frac{1}{100}\right)$. Estimate the error in the approximation.

Problem 7 Use the Taylor polynomial with $n = 4$ to approximate $e^{0.1}$. Estimate the error in the approximation.

Problem 8 Approximate $\int_0^{\frac{1}{10}} \frac{\sin x}{x}\, dx$ using the first three nonzero terms of a Taylor series, and estimate the error in the approximation.

Problem 9 Find the length of the parametric curve

$$x = 2t, \;\; y = t^3 + \frac{1}{3t}, \;\; 1 \le t \le 2.$$

Problem 10 Find the length of the curve $y = \frac{3}{8}(x^{4/3} - 2x^{2/3}), \ \ 1 \le x \le 8.$

Problem 11 Find the area of the region between the parametric curve $x = 4\cos t$, $y = 3\sin t,\ 0 \le t \le \pi,$ and the x –axis.

Problem 12 Let C be the parametric curve

$$x(t) = 2 + \cos t, \;\; y(t) = 1 - \sin t, \;\; 0 \leq t \leq \pi.$$

(a) Sketch the curve.

(b) Find the direct relation between x and y by eliminating the parameter t.

(c) Find the slope of the tangent line at $t = \frac{\pi}{4}$.

Problem 13 Let C be the parametric curve

$$x(t) = t$$
$$y(t) = \frac{1}{3}t^{3/2} - t^{1/2}, \quad 1 \leq t \leq 4.$$

(a) Find the length of the curve.

(b) Find the area of the surface obtained by revolving the curve C about the line $x = 1$.