Section 12.3 The Dot Product

Let $\vec{a} =< a_{1,}a_2, a_3 >$ and $\vec{b} =< b_{1,}b_2, b_3 >$.

Then, the **dot product** between $\vec{a}$ and $\vec{b}$ is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Example 1 Compute the dot product between $\vec{a} = < 3,2,-1 >$ and $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$

Properties of dot product

a. **Dot product result in a number** (Why?) (Sometimes. dot product is called a scalar product)

b. $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ (Important)

c. $\vec{a} \cdot \vec{0} = 0$

d. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

e. $\vec{a} \cdot (k\vec{b}) = k(\vec{a} \cdot \vec{b}) = k\vec{a} \cdot \vec{b}$, k a scalar

f. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

Let θ $(0 \leq \theta \leq \pi)$ be the angle between the representation of $\vec{a}$ and $\vec{b}$.

Then, we have

$$\vec{a} \cdot \vec{b} = |\vec{a}|\left|\vec{b}\right| \cos\theta$$

Observe that

a. $\vec{a} \cdot \vec{b} = 0$ if $\vec{a} \perp \vec{b}$ (**very important**)

b. $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\left|\vec{b}\right|}$

Example 2 Let $\vec{a} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{b} =< 1,1,-3 >$. Is $\vec{a} \perp \vec{b}$?

Projections:

Denote the **Vector Projection of** $\vec{b}$ **onto** $\vec{a}$ by **Proj**$_{\vec{a}}\vec{b}$ **.**

The length of the vector projection is called the **Scalar projection of** $\vec{b}$ **onto** $\vec{a}$ and is denoted by **Comp**$_{\vec{a}}\vec{b}$.

Illustration of the vector projection of $\vec{b}$ onto $\vec{a}$ when the angle θ between $\vec{a}$ and $\vec{b}$ is $0 \leq \theta \leq \pi$:

Direction of $\text{Proj}_{\vec{a}}\vec{b} = \frac{\vec{a}}{|\vec{a}|}$.

Length of $\text{Proj}_{\vec{a}}\vec{b}$(=Scalar projection of $\vec{b}$ onto $\vec{a}$ =$\text{Comp}_{\vec{a}}\vec{b}$) $= |b| \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$.

$\text{Proj}_{\vec{a}}\vec{b} = \text{Comp}_{\vec{a}}\vec{b}\ \frac{\vec{a}}{|\vec{a}|} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\vec{a}$

Scalar projection of $\vec{b}$ onto $\vec{a}$: $\quad \text{Comp}_{\vec{a}}\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

Vector projection of $\vec{b}$ onto $\vec{a}$ **:** $\quad \text{Proj}_{\vec{a}}\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\vec{a}$

Example 3 Let $\vec{a} =< -2, 1, 1 >$ and $\vec{b} =< 4, -3, 1 >$. Find $\text{Proj}_{\vec{a}}\vec{b}$.

Example 4 (Directional angle= angles($\in [0, \pi]$) with positive x-, y-, z-axes) Let $\vec{a} =< 2, 3, -6 >$. Find the angle between $x - axis$ and the vector $\vec{a}$.

HW: 1, 7, 9, 17, 25, 27, 29, 31, 41, 43.