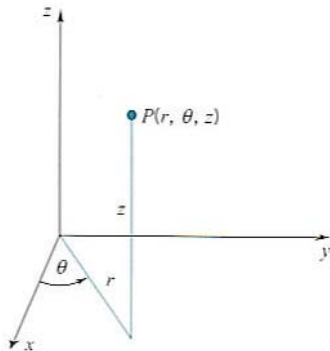


Section 12.7 Cylindrical and Spherical Coordinates

Cylindrical system = Polar coordinate + z



$$(x, y, z) \rightarrow (r, \theta, z)$$

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z$$

$$r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}$$

Example 1

a. Change $(2, \frac{\pi}{3}, 3)$ into the rectangular system.

b. Convert $(1, -1, 2)$ into cylindrical system.

Example 2 Sketch the graph.

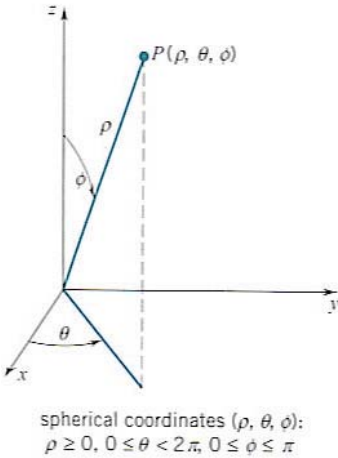
a. $z = r$

b. $r = 1$

d. $\theta = \frac{\pi}{4}$

Spherical system

$$(x, y, z) \rightarrow (\rho, \theta, \phi)$$
$$\rho \geq 0, \quad 0 \leq \phi \leq \pi.$$



θ : angle on xy -plane in usual way, ϕ : angle measured from positive z -axis

Conversion factors:

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \tan \theta = \frac{y}{x}, \quad \cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$$

Example 3 Convert $(1, 1, 1)$ in rectangular coordinates into spherical coordinates.

Example 4 Change $(2, \frac{\pi}{6}, \frac{\pi}{4})$ in spherical coordinates to rectangular coordinates.

Example 5 Sketch the graph.

a. $\rho = 2$

b. $\phi = \frac{\pi}{4}$

Example 6 Write the equation in rectangular coordinates.

a. $\rho = \cos \phi$

b. $\rho \sin \phi = 1$

HW: 3, 7, 9, 21, 31, 35, 37, 41, 61