

Section 13.1 Vector Functions and Space Curves

Let f, g, h be real-valued functions. And consider the vector

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}.$$

We call such a function a vector-valued function or **vector function**.

Example 1 Vector functions.

a. $\vec{r} : \mathbb{R} \rightarrow \mathbb{R}^2$ such that $\vec{r}(t) = \langle 2t + 1, t^2 \rangle$

b. $\vec{r} : \mathbb{R} \rightarrow \mathbb{R}^3$ such that $\vec{r}(t) = \langle 1 - 2t, 3 + 2t, 2 - t \rangle$

- **Limit of a vector function: take the limit componentwise.**

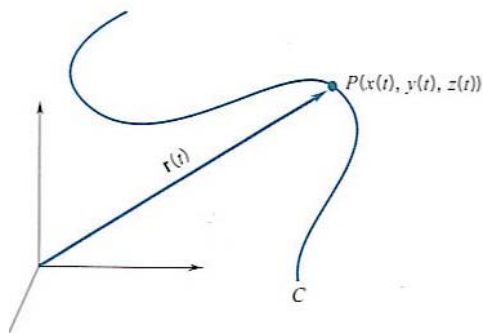
Example 2 Find $\lim_{t \rightarrow 0} \langle \cos t, \sin t, t \rangle$.

Vector function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ represents a curve in the space (space curve). We may think it as a trace of a moving particle (t : time)

Write $x = f(t), y = g(t), z = h(t)$.

Then it is said to be a **parametric representation of the curve** given by

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle.$$



$$C : \mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

Example 3 Sketch the curve with the following vector equation.

a. $\vec{r}(t) = \langle t + 1, t^2 \rangle$: plane curve

b. $\vec{r}(t) = \langle 1 + 2t, -2t, 2 - t \rangle$

c. $\vec{r}(t) = \langle 2 \cos t, 2 \sin t, t \rangle$

HW: 5, 7, 13, 15, 17, 19, 21.