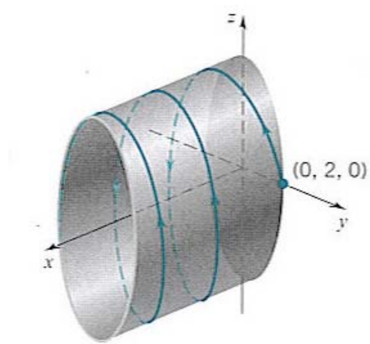
Section 13.2 Derivatives and Integrals of Vector Functions

- **Derivatives** $\vec{r}\,'(t)$ of vector function $\vec{r}(t) =< f(t),\ g(t),\ h(t) >$:
  $\vec{r}\,'(t) =< f'(t),\ g'(t),\ h'(t) >$ (Differentiate componentwise.)
- the **tangent vector** $\vec{r}\,'(t)$
- the **unit tangent vector** $\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$: indicates the direction of the curve
- the **tangent line to the curve** C **at** P : the line through $P = \vec{r}(t_0)$ parallel to the tangent vector $\vec{r}\,'(t_0)$.

Example 1 Let $\vec{r}(t) =< t,\ 2\cos t,\ 2\sin t >$.



a. Find $\vec{r}\,'(t)$.

b. Find unit tangent vector at $t = \frac{\pi}{4}$.

c. Find the parametric equation for the tangent line to $\vec{r}(t)$ at $t = \frac{\pi}{4}$

Differentiation Rules:

Let $\vec{u}(t)$, $\vec{v}(t)$ be differentiable vector functions, $f(t)$ be a real-valued function.
Then

a. $(\vec{u}(t) + \vec{v}(t))' = \vec{u}'(t) + \vec{v}'(t)$

b. $(f(t)\vec{u}(t))' = f'(t)\vec{u}(t) + f(t)\vec{u}'(t)$

c. $(\vec{u}(t) \cdot \vec{v}(t))' = \vec{u}'(t) \cdot \vec{v}(t) + \vec{u}(t) \cdot \vec{v}'(t)$ $\qquad (\vec{u}(t) \times \vec{v}(t))' = \vec{u}'(t) \times \vec{v}(t) + \vec{u}(t) \times \vec{v}'(t)$
(Product Rule)

d. $\left(\vec{u}(f(t))\right)' = f'(t)\vec{u}'(f(t))$ (Chain Rule)

Example 2 Show that if $\left|\overrightarrow{r(t)}\right| = c$ (a constant), then $\vec{r}'(t)$ is orthogonal to $\vec{r}(t)$.

- **the angle of intersection of two curves**: the angle between the tangent vectors at the point of intersection.

Example 3 The curves $r_1(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\vec{k}$ and $\vec{r_2}(u) = (1+u)\vec{i} + u^2\vec{j} + u^3\vec{k}$ intersect at $(1,\ 0,\ 0)..$
Find the angle of intersection of these curves.

- **Integrals:** $\int\left(f\vec{i}+g\vec{j}+h\vec{k}\right)dt=\left(\int fdt\right)\vec{i}+\left(\int gdt\right)\vec{j}+\left(\int hdt\right)\vec{k}$. (integrate componentwise.)

Example 4 Evaluate $\int_0^1\left(\vec{i}+t\vec{j}+t^2\vec{k}\right)dt$ and $\int\left(\vec{i}+t\vec{j}+t^2\vec{k}\right)dt$

HW: 17, 19, 21, 23, 25, 31, 37

note: $\int \ln t dt = t\ln t - t,$ using integration by parts $\int udv = uv - \int vdu$ with $u = \ln t,\ dv = dt.$