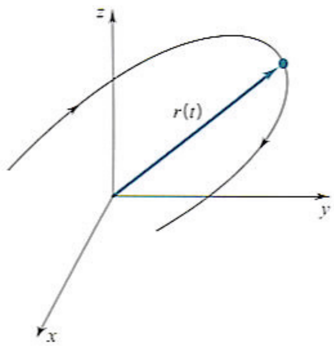
Section 13.4 Motion in Space: Velocity and Acceleration
Remember that $\vec{r}(t) =< x(t), y(t), z(t) >$ represent a curve in $\mathbb{R}^3$. We may also think

$\vec{r}(t) = (x(t), y(t), z(t))$ as a trace of a particle.



- **position vector** $\vec{r}(t)$
- **velocity vector** $\vec{v}(t) = \vec{r}^{\,\prime}(t)$
- **speed** $\left|\vec{v}(t)\right| = \left|\vec{r}^{\,\prime}(t)\right|$
- **acceleration vector** $\vec{a}(t) = \vec{r}^{\,\prime\prime}(t)$

Example 1 A particle moves along $\vec{r}(t) =< 2, t^2, (t-1)^2 >$.
a. Find the velocity, speed and acceleration.

b. When is the speed minimum?

Example 2 A particle has a velocity $\vec{v}(t) =< -\sin t, 1, e^{-t} >$ and its initial position is at $\vec{r}(0) =< 0, 2, 1 >$. Find its position vector.

Newton's Second Law of Motion: $\vec{F}(t) = m\vec{a}(t)$, $\vec{F}$: a force, m : mass, $\vec{a}$: acceleration

Example 3 Find the force required so that a particle of mass m has the position vector $\vec{r}(t) =< 0,\ t,\ t^2 >$.

Tangential and Normal components of Acceleration

- $\vec{a} = a_T\vec{T} + a_N\vec{N},$

where a_T is the tangential component, a_N is the normal component.

- $a_T = \vec{T} \cdot \vec{a} = \frac{\vec{r}' \cdot \vec{r}''}{|\vec{r}'|},$ $\quad a_N = \left|\vec{T} \times \vec{a}\right| = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|}.$

Example 4 A particle moves with position function $\vec{r}(t) =< e^t, e^{-t}, 0 >$. Find the tangential and normal componets of acceleration.

HW: 11, 15, 19, 20, 21, 22, 33
hint: the problem 22 is very similar to the example 2 of section 13.2 in the lecture note.