

Section 14.1 Functions of Several variable

Real-valued function means "you get a number."

Example 1 Examples of real-valued function.

a. $f(x,y) = xy \quad f: \mathbb{R}^2 \rightarrow \mathbb{R}$ (We often write it as $z = f(x,y)$, i.e., $z = xy$ in this case.)

b. $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right) \quad f: \mathbb{R}^2 \rightarrow \mathbb{R}$

c. $f(x,y,z) = x^2 + y \tan z \quad f: \mathbb{R}^3 \rightarrow \mathbb{R}$ (often write it as $w = f(x,y,z)$, i.e., $w = x^2 + y \tan z$ in this case.)

Domain:

$$\frac{f}{g} : g \neq 0, \quad \sqrt{\text{positive or } 0}, \quad \ln(\text{positive})$$

Example 2 Find and sketch the domain of $f(x,y) = \ln(x - 3y)$.

Example 3 Find the domain and range, and sketch the graph of $g(x,y) = \sqrt{9 - x^2 - y^2}$

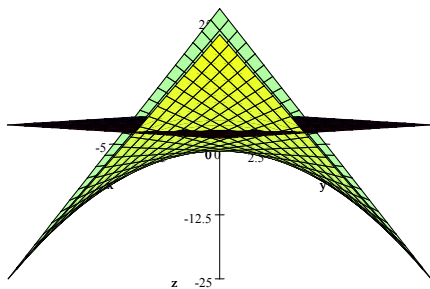
Level(contour) Curves and Level Surfaces

Let $z = f(x,y)$. Then,

- **Level curves** are defined by $k = f(x,y)$ for different values of k .

Example 4 Sketch level curves of

$$g(x,y) = xy$$



Graphs

Example 5 Sketch the graph of $f(x,y) = 6 - 3x - 2y$.

Let $w = f(x,y,z)$.

● **Level surfaces** are defined by $k = f(x,y,z)$ for different values of k .

Example 6 Sketch level surfaces of $g(x,y,z) = \sqrt{x^2 + y^2 + z^2}$.

HW: 8, 11, 13, 23, 29, 39, 59(do it for $k=0, -1, 1$).