

Section 14.2 Limits and Continuity

Review: Let $y = f(x)$.

a. $\lim_{x \rightarrow a} f(x)$ exists if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

b. $f(x)$ is continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$ That is, the limit value is same as the function value at $x = a$.

Example 1 Let $f(x) = \frac{|x|}{x}$. Show that $\lim_{x \rightarrow 0} f(x)$ does not exist.

Review: Theorem on continuous function (true for \mathbb{R}^n)

- continuous+continuous=continuous
- continuous·continuous=continuous
- composite of continuous functions= continuous function
- $\frac{\text{continuous}}{\text{continuous}} = \text{continuous}$ if denominator $\neq 0$

Examples of continuous functions (true for \mathbb{R}^n)

- polynomial
- rational if denominator $\neq 0$
- $\sin x, \cos x, e^x, \ln x, \dots$

Limits in \mathbb{R}^3 : Let $z = f(x, y)$.

- $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$ exists if $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$ exists and **same in any direction**.

Example 2 Let $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$.

a. $\lim_{(x,y) \rightarrow (1,2)} f(x, y) =$

b. Show that $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist

Example 3 Let $f(x, y) = \frac{xy + y^3}{x^2 + y^2}$

a. $\lim_{(x,y) \rightarrow (0,1)} f(x, y) =$

b. Show that $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist

Example 4 Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^4+y^2}$ does not exist

Example 5 (Very useful) Compute $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^2+y^2}$ by using polar coordinates.

● **A function f of two variables is continuous at (a, b) if**

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = f(a,b).$$

Example 6 Let $f(x,y) = \begin{cases} \frac{2xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

- Is $f(x,y)$ continuous other than $(0,0)$?
- Is $f(x,y)$ continuous at $(0,0)$?

HW: 5, 7, 11(use polar coordinates), 13, 15(rationalize the denominator), 17, 37.