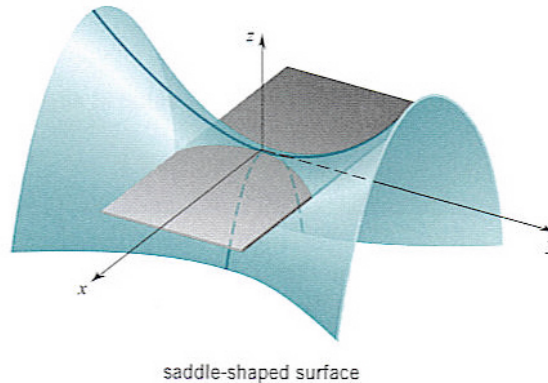
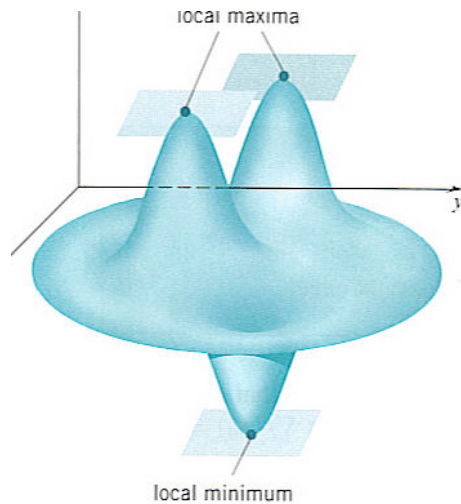


Section 14.7 Maximum and Minimum Values

● Local maximum and local minimum

We say $f(x,y)$ has a local maximum at (a,b) if $f(a,b) \geq f(x,y)$ for all (x,y) around (a,b) .

We say $f(x,y)$ has a local minimum at (a,b) if $f(a,b) \leq f(x,y)$ for all (x,y) around (a,b) .

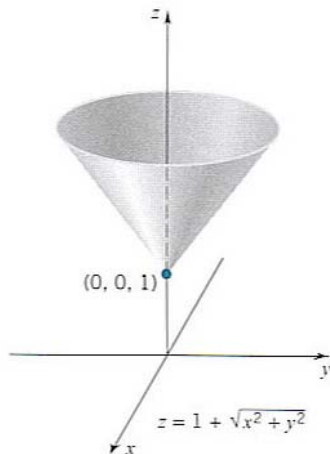


- **Theorem:** If f has a local maximum or minimum at (a,b) and the first partial derivatives exist there, then $f_x(a,b) = f_y(a,b) = 0$, i.e., $\vec{\nabla}f(a,b) = \vec{0}$.

The geometric interpretation of the above theorem: if the graph of f has a local maximum or minimum, then the tangent plane must be horizontal.

- **Critical point:** (a,b) is called a critical point of a function if $f_x(a,b) = 0$ and $f_y(a,b) = 0$, or either one of $f_x(a,b), f_y(a,b)$ does not exist.

Example 1 Find all the critical points of $f(x,y) = x^4 + y^4 - 4xy + 1$.



Example 2 Let $f(x,y) = 1 + \sqrt{x^2 + y^2}$. Then, f_x and f_y are not defined at $(0,0)$. So, $(0,0)$ is a critical point and in fact, $f(0,0) = 1$ is obviously a local minimum.

● **Second Derivatives Test:**

Let (a,b) be a critical point and assume that the second partial derivatives are continuous on a disk centered at (a,b) .

Let

$$\mathbf{D} = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2.$$

- a. If $D > 0$ at (a,b) and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.
- b. If $D > 0$ at (a,b) and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.
- c. If $D < 0$ at (a,b) , then $f(a,b)$ is not a local maximum or a local minimum. $((a,b)$ is called a saddle point and the graph of f crosses its tangent plane at (a,b)).
- d. If $D = 0$, then the theorem does not say anything.

Example 3 Find local extreme values of $f(x,y) = x^4 + y^4 - 4xy + 1$.

Example 4 Find local extreme values of $f(x,y) = 1 + 2xy - x^2 - y^2$.

- **Absolute Maximum and Minimum Values**

A closed set in R^2 is one that contains all its boundary. A bounded set in R^2 is one that is contained within some disk, that is, it is finite in extent.

- **Extreme Value Theorem for Functions of Two variables:**

A continuous function on a closed and bounded set D must have absolute maximum and absolute minimum in D .

How to find absolute maximum and minimum?

1. **Find the values of f at the critical points of f in D .**
2. **Find the extreme values of f on the boundary of D .**
3. **The largest value from step 1 and step 2 is the absolute maximum; the smallest one is the absolute minimum.**

Example 5 Find the absolute maximum and minimum values of $f(x,y) = xy - 2x - 3y$ on the closed triangular region with vertices $(0,0)$, $(4,8)$ and $(4,0)$.

Example 6 Find the maximum volume of a rectangular box given that the sum of the lengths of edges is 24.

HW: 5, 7, 9, 27, 29, 31, 33, 39, 41, 45, 49