

Section 15.5 Applications of Double Integrals

Density and Mass:

Consider a flat plate (called a lamina) which occupies the region D . Suppose that its density at a point (x, y) in D is $\rho(x, y)$.

Then the **total mass** m is

$$m = \iint_D \rho(x, y) \, dA.$$

Remark: If we consider an electric charge distributed over the region D and $\sigma(x, y)$ is the charge density at (x, y) , then the total charge is

$$Q = \iint_D \sigma(x, y) \, dA.$$

Moments and Centers of Mass:

The **moment about x -axis** is

$$M_x = \iint_D y \rho(x, y) \, dA.$$

The **moment about y -axis** is

$$M_y = \iint_D x \rho(x, y) \, dA.$$

Let (\bar{x}, \bar{y}) be the **center of mass**, i.e., a lamina balances horizontally at (\bar{x}, \bar{y}) .

Then,

$$\bar{x} = \frac{1}{m} \iint_D x \rho(x, y) \, dA, \quad \bar{y} = \frac{1}{m} \iint_D y \rho(x, y) \, dA$$

Example 1 Find the mass and center of mass if a triangular lamina with vertices $(0, 0)$, $(1, 0)$, $(0, 2)$ if the density function is $\rho(x, y) = 1 + 3x + y$.

Example 2 The density at any point on a semicircular lamina is inversely proportional to the distance from the center of the circle. Find the center of mass of the lamina.

HW: 5, 7, 9, 11, 14

$$\text{Ans for 14: } (\bar{x}, \bar{y}) = \left(0, \frac{3\sqrt{3}}{2(3\sqrt{3}-\pi)} \right)$$