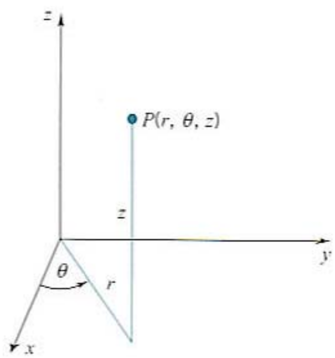
Section 12.7 & 15.8 Cylindrical and Spherical Coordinates & Triple Integrals in Cylindrical and Spherical Coordinates

Cylindrical Coordinates = **Polar coordinate** $+z$



$(x,y,z) \rightarrow (r,\theta,z)$
- $x = r\cos\theta,\ y = r\sin\theta,\ z = z$
- $r^2 = x^2 + y^2,\ \tan\theta = \frac{y}{x}$

Note that (typically) $r \geq 0$ and $0 \leq \theta \leq 2\pi$.

Example 1
a. Change $(2, \frac{\pi}{3}, 3)$ into the rectangular system.

b. Convert $(1, -1, 2)$ into cylindrical system.

Example 2 Sketch the graph.
a. $z = r$

b. $r = 1$

d. $\theta = \frac{\pi}{4}$

$$\iiint f(x,y,z) = \iiint f(r\cos\theta, r\sin\theta, z)\ r dz dr d\theta$$

In cylindrical coordinates, $dV = r dz dr d\theta$

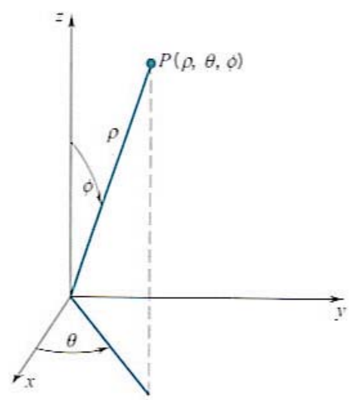
Example 3 Set up the triple integral for the volume of the solid bounded by $z = 4 - x^2 - y^2$ and $z = 0$ in cylindrical coordinates.

Example 4 Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\ dz dx dy$.

Spherical Coordinates

$(x, y, z) \to (\rho, \theta, \phi)$

$\rho \geqslant 0, \quad 0 \leq \phi \leq \pi.$



spherical coordinates (ρ, θ, ϕ):
$\rho \geq 0, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi$

θ : angle on xy –plane in usual way, ϕ : angle measured from positive z-axis

- $x = \rho \sin\phi \cos\theta, \; y = \rho \sin\phi \sin\theta, \; z = \rho \cos\phi$
- $\rho^2 = x^2 + y^2 + z^2, \; \tan\theta = \dfrac{y}{x}, \quad \cos\phi = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

Note that $\rho \geq 0, \; 0 \leq \phi \leq \pi, \; 0 \leq \theta \leq 2\pi$ and $r = \rho \sin\phi$.

Example 5 Convert $(1, 1, 1)$ in rectangular coordinates into spherical coordinates.

Example 6 Change $(2, \frac{\pi}{6}, \frac{\pi}{4})$ in spherical coordinates to rectangular coordinates.

Example 7 Sketch the graph.

a. $\rho = 2$

b. $\phi = \frac{\pi}{4}$

Example 8 Write the equation in rectangular coordinates.
a. $\rho = \cos\phi$

b. $\rho\sin\phi = 1$

$$\boxed{\iiint f(x,y,z)\ dV = \iiint f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\ \rho^2\sin\phi\ d\rho d\phi d\theta}$$

In spherical coordinates, $dV = \rho^2\sin\phi\ d\rho d\phi d\theta$.

Example 9 Convert $\iiint_B e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\ dV$ to spherical coordinates, where B is the unit ball B : $x^2 + y^2 + z^2 \le 1$.

Example 10 Set up the triple integral for the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$ in spherical coordinates (see the figure 10 on page 1022 in the textbook).

Example 11 Convert the integral $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2) dz dx dy$ to spherical coordinates.

HW:
Section 12.7: 3, 7, 9, 21, 31, 35, 37, 41, 61
Section 15.8: 1, 3, 7, 11, 13, 17, 21, 23, 33, 35