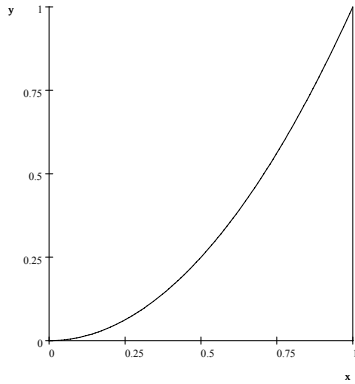


c) $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx dy$ (Hint: Change the order of integration.)

Note that the region of integration is $\{(x,y) : \sqrt{y} \leq x \leq 1, 0 \leq y \leq 1\}$. Sketching the region, we have

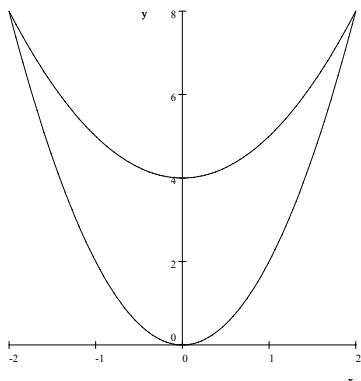


Reading off the region of integration again, we see that $0 \leq y \leq x^2$, $0 \leq x \leq 1$. Hence,

$$\begin{aligned} \int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx dy &= \int_0^1 \int_0^{x^2} e^{x^3} dy dx \\ &= \int_0^1 [ye^{x^3}]_{y=0}^{y=x^2} dx = \int_0^1 x^2 e^{x^3} dx \\ &= \int_{x=0}^{x=1} \frac{1}{3} e^u du, \quad u = e^{x^3}, \quad du = 3x^2 dx \\ &= \left[\frac{1}{3} e^u \right]_{x=0}^{x=1} = \left[\frac{1}{3} e^{x^3} \right]_{x=0}^{x=1} = \frac{1}{3} (e - 1). \end{aligned}$$

Note: In what follows, I will denote regions in plane by D and solids in space by E .

Problem 2 Express the area of the region bounded by $y = 4 + x^2$ and $y = 2x^2$ as an iterated double integral. Do not evaluate.

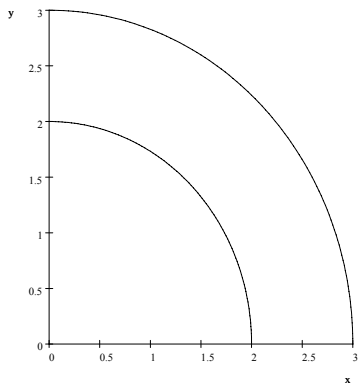


Intersections: $4 + x^2 = 2x^2 \Rightarrow x = -2, 2$. So we can express the region as $\{(x, y) : -2 \leq x \leq 2, 2x^2 \leq y \leq 4 + x^2\}$.

Area of the region D is $\iint_D dA$. So,

$$A(D) = \int_{-2}^2 \int_{2x^2}^{4+x^2} dy dx.$$

Problem 3 Express the area of the region in the first quadrant bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ as an iterated double integral in polar coordinates. Do not evaluate.

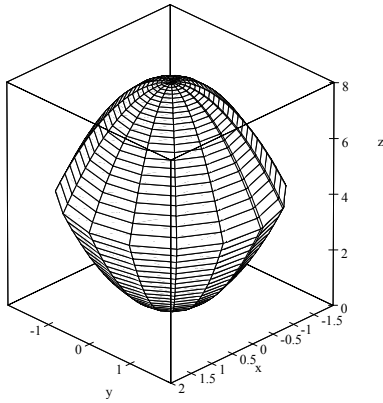


The region can be expressed as $\{(r, \theta) : 2 \leq r \leq 3, 0 \leq \theta \leq \frac{\pi}{2}\}$. Note that the area element dA becomes $rdrd\theta$ in polar coordinates.

So,

$$A(D) = \int_0^{\frac{\pi}{2}} \int_2^3 r dr d\theta.$$

Problem 4 Express the volume of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$ as an iterated **triple integral**. *Do not evaluate.*



The projection of the solid onto xy -plane is $\{(x,y) : x^2 + y^2 \leq 4\}$, since $x^2 + y^2 = 8 - x^2 - y^2 \Rightarrow x^2 + y^2 = 4$.

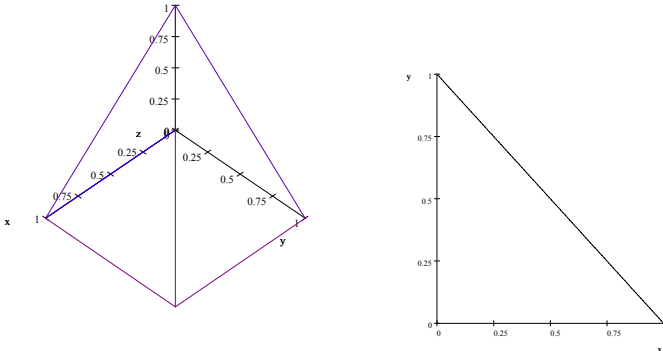
So, the region E is

$$\{(x,y,z) : -2 \leq x \leq 2, -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}, x^2 + y^2 \leq z \leq 8 - x^2 - y^2\}$$

Also note that the volume of the solid E is $V(E) = \iiint_E dV$. Hence,

$$V(E) = \iiint_E dV = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} dz dy dx.$$

Problem 5 Express $\iiint_E x dV$ as an iterated integrals, where E is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $y + z = 1$, and $x + z = 1$. Do not evaluate.



Projection onto yz -plane: $\{(y,z) : 0 \leq y \leq 1 - z, 0 \leq z \leq 1\}$

We will project the solid onto yz -plane. Then we get the region bounded by the lines $x = 0$, $y = 0$, and $y + z = 1$. Since we project the region onto yz -plane, we need to determine the range for x .

We see from the graph (or the equations given) that $0 \leq x \leq 1 - z$.

Hence, the solid is

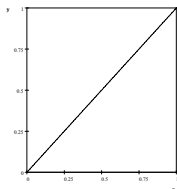
$$\{(x,y,z) : 0 \leq x \leq 1 - z, 0 \leq y \leq 1 - z, 0 \leq z \leq 1\}.$$

Thus,

$$\iiint_E x dV = \int_0^1 \int_0^{1-z} \int_0^{1-z} x dx dy dz.$$

Remark: Several different answers are possible in this problem: if you project onto xz -plane, one possible solution is $\int_0^1 \int_0^{1-z} \int_0^{1-z} x dy dx dz$.

If you project onto xy -plane, it gets complicated. We need to divide the square on xy -plane into upper half triangle and lower half triangle.



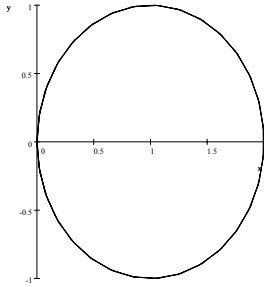
The lower half triangle is the projection of the plane $x + z = 1$ and upper half triangle is the projection of the plane $y + z = 1$. So,

$$\iiint_E dV = \int_0^1 \int_y^1 \int_0^{1-x} x dz dx dy + \int_0^1 \int_0^y \int_0^{1-y} x dz dx dy.$$

Also, we should be careful choosing the right projection: for example, if we mistakenly use the lower triangle for the projection of the plane $y + z = 1$, we get $\int_0^1 \int_y^1 \int_0^{1-y} x dz dx dy$.

But $\int_0^1 \int_0^y \int_0^{1-y} x dz dx dy = \frac{1}{24}$, while $\int_0^1 \int_y^1 \int_0^{1-y} x dz dx dy = \frac{5}{24}$.

Problem 6 Express the surface area of the part of the upper-hemi sphere $z = \sqrt{4 - x^2 - y^2}$ that lies within the cylinder $x^2 + y^2 = 2x$ as an iterated double integral **in polar coordinates**. Do not evaluate.



Recall the surface area formula: $A(S) = \iint_D \sqrt{1 + f_x^2 + f_y^2} dA$.

Here, .

$$f(x,y) = \sqrt{4 - x^2 - y^2} \Rightarrow f_x = \frac{-x}{\sqrt{4 - x^2 - y^2}}, f_y = \frac{-y}{\sqrt{4 - x^2 - y^2}} \Rightarrow f_x = -\frac{r \cos \theta}{\sqrt{4 - r^2}}, f_y = -\frac{r \sin \theta}{\sqrt{4 - r^2}}.$$

Note that $x^2 + y^2 = 2x \Rightarrow r^2 = 2r \cos \theta \Rightarrow r = 2 \cos \theta$.

So, in polar coordinates, we write the domain D as

$$D = \left\{ (r, \theta) : 0 \leq r \leq 2 \cos \theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right\}.$$

Hence, the surface area is

$$\begin{aligned} A(S) &= \iint_D \sqrt{1 + f_x^2 + f_y^2} dA \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} \sqrt{1 + \frac{r^2 \cos^2 \theta}{4 - r^2} + \frac{r^2 \sin^2 \theta}{4 - r^2}} r dr d\theta \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} \sqrt{\frac{4 - r^2 + r^2 \cos^2 \theta + r^2 \sin^2 \theta}{4 - r^2}} r dr d\theta \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} \sqrt{\frac{4}{4 - r^2}} r dr d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} \frac{2}{\sqrt{4 - r^2}} r dr d\theta. \end{aligned}$$

