Practice Problems for the Final

Problem 1 Let $\vec{a} =< 1,0,1 >$, $\vec{b} =< 0,1,1 >$ and $\vec{c} =< 1,1,1 >$.

a) Compute the angle between $\vec{a}$ and $\vec{b}$.

b) Find a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$.

c) Find (if possible) a vector $\vec{d}$ so that $\vec{b} \cdot \vec{c} = \vec{d}$. If there is none , explain your answer.

Problem 2 Find the parametric equation of the line which is perpendicular to the plane $x + 2y + 3z = 4$ and passes through $(-1,-2,-3)$.

Problem 3 Let $F(x,y,z) = xe^{yz}$

a) Find the gradient of F at $(-2, 1, 1)$.

b) Find the directional derivative of F in the direction of $\vec{v} =< 1, -2, 3 >$ at $(-2, 1, 1)$.

c) Find the equation of the tangent plane to the surface $xe^{yz} = -2e$ at $(-2, 1, 1)$

d) Find the rate of change of z with respect to y if $xe^{yz} = -2e$.

Problem 4 Let $f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & \text{if } (x,y) = (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$.

a) Compute $\lim_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$ by using polar coordinates.

b) Is $f(x,y)$ continuous at $(0,0)$? Explain your answer.

Problem 5 Find the absolute maximum and minimum values of $f(x,y) = 2x^2 + x + y^2 - 2$ on the region $D = \{(x,y) |\ x^2 + y^2 \leq 4\}$

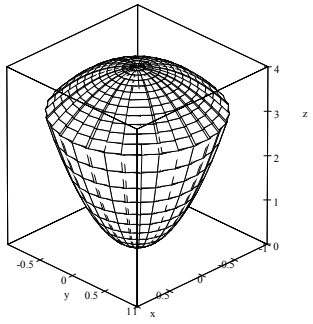
Problem 6

a) Evaluate the integral $\int_0^1 \int_0^x y\sqrt{x^2+y^2}\,dydx$

b) Evaluate $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 dxdy$ by converting to polar coordinates.

Problem 7

a) Express the volume of the solid above the surface $z = 3x^2 + 3y^2$ and below the surface $z = 4 - x^2 - y^2$ in cylindrical coordinates. Do not evaluate.



b) Express the volume of the sphere $\rho = 1$ in spherical coordinates. Do not evaluate.

Problem 8

a) Convert $\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}}$ *dzdydx* to cylindrical coordinates.

b) Convert $\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}}$ *dzdydx* to spherical coordinates.

Problem 9 Evaluate $\iint_D xydxdy,$ where D is the region in the first quadrant bounded by the curves $x^2 + y^2 = 4,\ x^2 + y^2 = 9, x^2 - y^2 = 1, x^2 - y^2 = 4.$