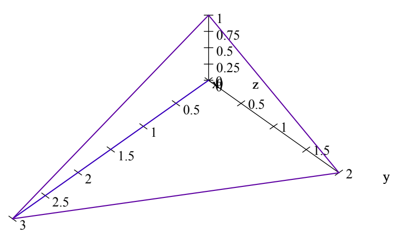
MATH 242 QUIZ 10

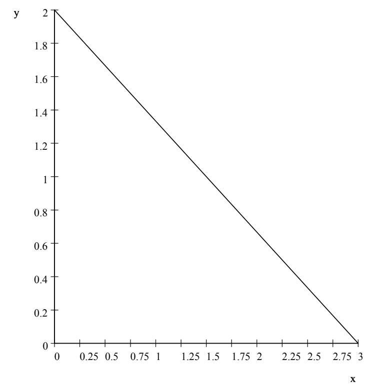
NAME (Print your name): Type A

SECTION:

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

Problem 1(12 pt) Use a triple integral to find the volume of the tetrahedron bounded by the four planes $x = 0, y = 0, z = 0$, and $\frac{x}{3} + \frac{y}{2} + z = 1$.





$$\int_0^3\int_0^{2-\frac{2}{3}x}\int_0^{1-\frac{1}{3}x-\frac{1}{2}y} dzdydx = \int_0^3\int_0^{2-\frac{2}{3}x}\left(1-\frac{1}{3}x-\frac{1}{2}y\right)dydx$$

$$= \int_0^3\left[y-\frac{1}{3}xy-\frac{1}{4}y^2\right]_{y=0}^{2-\frac{2}{3}x}dx$$

$$= \int_0^3\left(2-\frac{2}{3}x-\frac{1}{3}x\left(2-\frac{2}{3}x\right)-\frac{1}{4}\left(2-\frac{2}{3}x\right)^2\right)dx$$

$$= \int_0^3\left(2-\frac{2}{3}x\right)dx-\frac{1}{3}\int_0^3\left(2x-\frac{2}{3}x^2\right)dx-\frac{1}{4}\int_0^3\left(2-\frac{2}{3}x\right)^2dx$$

$$= \left[2x-\frac{1}{3}x^2\right]_{x=0}^3-\frac{1}{3}\left[x^2-\frac{2}{9}x^3\right]_{x=0}^3-\frac{1}{4}\int_{x=0}^{x=3}u^2\left(-\frac{3}{2}\right)du,\quad u=2-\frac{2}{3}x$$

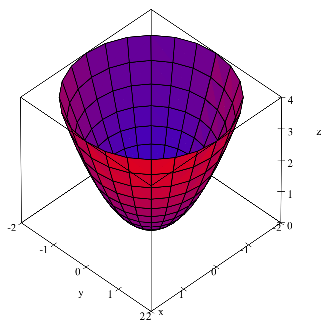
$$= 6-3-\frac{1}{3}(9-6)+\left[\frac{1}{8}u^3\right]_{x=0}^{x=3}$$

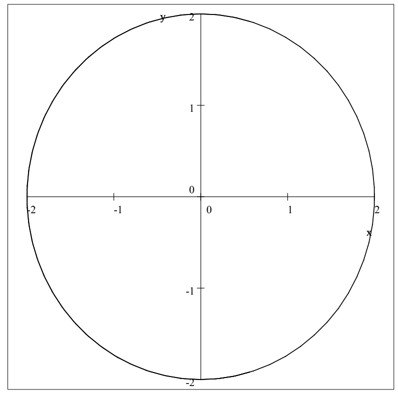
$$= 2+\frac{1}{8}\left[\left(2-\frac{2}{3}x\right)^3\right]_{x=0}^{x=3}$$

$$= 2+\frac{1}{8}(0-8) = 1.$$

Remark: Using the volume formula, the volume of tetrahedron is $\frac{1}{3}$(area of base)(height)=$\frac{1}{3}\cdot\frac{1}{2}\cdot 3\cdot 2\cdot 1 = 1$.

Problem 2(8 pt) Express the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$ as an **iterated double integral in polar coordinates**. *Do not evaluate.*





$f(x,y) = x^2 + y^2$. So, $f_x = 2x$, $f_y = 2y$.

Note that $dA = r dr d\theta$ in polar coordinates.
Using the surface area formula, we get

$$S = \iint_D \sqrt{1 + 4x^2 + 4y^2 dA}$$
$$= \int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2}\, r dr d\theta.$$