

MATH 242 QUIZ 2

NAME (Print your name): Type A

You need to show all your work to get a full credit.

Problem 1 a)(4 points) Find the point where the line $x = 2$, $y = -1 + 2t$, $z = 4 - t$ intersects the xy -plane.

solution) Since $z = 0$ on the xy -plane, $z = 4 - t = 0$.

$$t = 4.$$

Hence,

$$x = 2, \quad y = -1 + 2(4) = 7.$$

Thus, the given line intersects xy -plane on $(2, 7, 0)$.

b)(6 points) Find a parametric equation for the line that passes through the points $(2, 3, 3)$ and is parallel to the line $x = 2$, $y = -1 + 2t$, $z = 4 - t$.

solution) A directional vector for the line $x = 2$, $y = -1 + 2t$, $z = 4 - t$ is $\langle 0, 2, -1 \rangle$. Since the line is parallel to the given line, $\langle 0, 2, -1 \rangle$ is also a directional vector for the line. So, the parametric equations for the line is

$$x = 2 + 0t = 2, \quad y = 3 + 2t, \quad z = 3 - t.$$

Problem 2(10 points) Find an equation for the plane that passes through $(1, -3, 5)$ and contains the line $x = -1 + 3t$, $y = -2 + t$, $z = 2 + 4t$.

solution) We need to find two nonparallel vectors \vec{a} , \vec{b} on the plane. Note that if $t = 0$, $x = -1$, $y = -2$, $z = 2$.

Possible choices are

$$\vec{a} = \langle 1 - (-1), -3 - (-2), 5 - 2 \rangle = \langle 2, -1, 3 \rangle,$$

$$\vec{b} = \text{directional vector of the line} = \langle 3, 1, 4 \rangle.$$

A normal vector the plane is

$$\begin{aligned} \vec{n} = \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 3 & 1 & 4 \end{vmatrix} \\ &= \begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} \vec{k} = -7\vec{i} + \vec{j} + 5\vec{k}. \end{aligned}$$

Hence, the equation of the plane is

$$-7(x - 1) + (y + 3) + 5(z - 5) = 0.$$

$$7x - y - 5z = -15.$$