MATH 242 QUIZ 4

NAME (Print your name): Type A

SECTION:

You need to show all your work to get a full credit.

Problem 1 Let $\vec{r}(t) = \cos 4t\vec{i} + 3t\vec{j} + \sin 4t\vec{k}$.

a)(8 points) Find the unit tangent and the unit normal vector.

solution) $\vec{r}'(t) = -4\sin 4t\vec{i} + 3\vec{j} + 4\cos 4t\vec{k}$

$|\vec{r}'(t)| = \sqrt{16+9} = 5$

$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{1}{5}(-4\sin 4t\vec{i} + 3\vec{j} + 4\cos 4t\vec{k})$

$\vec{T}'(t) = \frac{1}{5}(-16\cos 4t\vec{i} - 16\sin 4t\vec{k})$

$|\vec{T}'(t)| = \frac{16}{5}$

So, $\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = -\cos 4t\vec{i} - \sin 4t\vec{k}$.

b)(4 points) Find the normal vector to the osculating plane at $t = \dfrac{\pi}{12}$.

solution)The normal vector to the osculating plane at $t = \dfrac{\pi}{12}$ is

$$\begin{aligned}\vec{B}(\frac{\pi}{12}) &= \vec{T}(\frac{\pi}{12}) \times \vec{N}(\frac{\pi}{12}) \\ &= < -\frac{2\sqrt{3}}{5}, \frac{3}{5}, \frac{2}{5} > \times < -\frac{1}{2}, 0, -\frac{\sqrt{3}}{2} > \\ &= -\frac{3\sqrt{3}}{10}\vec{i} - \frac{4}{5}\vec{j} + \frac{3}{10}\vec{k}.\end{aligned}$$

Problem 2(8 points) A particle has a velocity $\vec{v}(t) = < 2e^t, 2e^{-t}, t >$ and its initial position is at $\vec{r}(0) = < 1, 0, 0 >$. Find its position vector.

solution)

$\vec{r}(t) = \int \vec{v}(t)dt = < 2e^t + C_1, -2e^{-t} + C_2, \frac{t^2}{2} + C_3 >$

$\vec{r}(0) = < 2 + C_1, -2 + C_2, C_3 > = < 1, 0, 0 >$

$C_1 = -1, C_2 = 2, C_3 = 0$

So, $\vec{r}(t) = < 2e^t - 1, -2e^{-t} + 2, \frac{t^2}{2} >$.