

## MATH 242 QUIZ 5

NAME (Print your name): Type A

### SECTION:

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

**Problem 1**(4 pt each) Let  $f(x, y) = (1 + x + y^2)^{4/3}$ . Find the following partial derivatives.

a)  $\frac{\partial f}{\partial x} = \frac{4}{3}(1 + x + y^2)^{\frac{1}{3}}$

b)  $\frac{\partial f}{\partial y} = \frac{4}{3}(1 + x + y^2)^{\frac{1}{3}} \cdot 2y = \frac{8}{3}y(1 + x + y^2)^{\frac{1}{3}}$

c)  $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{4}{9}(1 + x + y^2)^{-\frac{2}{3}} \cdot 2y$

d)  $\frac{\partial^2 f}{\partial y^2}$  Using the product rule, we have,

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) = \frac{8}{3}(1 + x + y^2)^{\frac{1}{3}} + \frac{8}{9}y(1 + x + y^2)^{-\frac{2}{3}} \cdot 2y$$

**Problem 2**(4 pt) Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$  does not exist.

Along the line  $y = mx$ , we have

$$\lim_{x \rightarrow 0} \frac{x^2}{x^2 + m^2 x^2} = \lim_{x \rightarrow 0} \frac{x^2}{x^2(1 + m^2)} = \frac{1}{1 + m^2}$$

which yields a different answer for different values of  $m$ . Hence the limit does not exist.