

## MATH 242 QUIZ 6 *Solutions*

NAME (Print your name): Type A

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

**Problem 1**(6 pt) Find the linear approximation of  $f(x, y) = ye^x$  at  $(0, 2)$  **and** use it to approximate  $f(0.1, 2.1)$ .

$$L(x, y) = f(0, 2) + f_x(0, 2)x + f_y(0, 2)(y - 2)$$

$$f_x = ye^x, \quad f_y = e^x$$

$$f(0, 2) = 2, \quad f_x(0, 2) = 2, \quad f_y(0, 2) = 1$$

$$f(x, y) \simeq 2 + 2(x - 0) + 1(y - 2)$$

$$f(0.1, 2.1) = 2 + 2(.1) + 1(.1) = 2.3$$

**Problem 2**(7 pt) Let  $z = xy + y^2$ , where  $x = r \cos \theta$  and  $y = r \sin \theta$ . Use the chain rule to find  $\frac{\partial z}{\partial \theta}$ .

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial \theta}$$

$$\frac{\partial x}{\partial \theta} = -r \sin \theta$$

$$\frac{\partial y}{\partial \theta} = r \cos \theta$$

$$\frac{\partial z}{\partial \theta} = y(-r \sin \theta) + (x + 2y)r \cos \theta$$

**Problem 3**(7 pt) Find the directional derivative of  $f(x, y) = \frac{x}{y}$  at  $(6, -2)$  in the direction of  $\langle -1, 3 \rangle$ .

$$\begin{aligned}\nabla f &= \left\langle \frac{1}{y}, \frac{-x}{y^2} \right\rangle \\ \nabla f(1, -3) &= \left\langle \frac{-1}{2}, \frac{-6}{4} \right\rangle = \frac{-1}{2} \langle 1, 3 \rangle \\ \vec{u} &= \frac{1}{\sqrt{10}} \langle -1, 3 \rangle \\ D_{\vec{u}} f(6, -2) &= \frac{-1}{2} \langle 1, 3 \rangle \cdot \frac{1}{\sqrt{10}} \langle -1, 3 \rangle \\ &= \frac{-1}{2\sqrt{10}}(-1 + 5) = \frac{-8}{2\sqrt{10}} = \frac{-4}{\sqrt{10}}\end{aligned}$$