

MATH 242 QUIZ 7

NAME (Print your name): Type A

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

Problem 1(8 pt) Find the equation of the tangent plane to the surface $xy^2z^3 = 12$ at $(3, 2, 1)$.

solution) A normal vector to the tangent plane at the point (x, y, z) is

$$\vec{\nabla}F(x, y, z) = \langle F_x, F_y, F_z \rangle = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle,$$
 where $F(x, y, z) = xy^2z^3$.

So, at $(3, 2, 1)$, we have

$$\vec{\nabla}F(3, 2, 1) = \langle 4, 12, 36 \rangle .$$

Hence, the equation of the tangent plane to the given surface at $(3, 2, 1)$ is

$$\begin{aligned} 4(x - 3) + 12(y - 2) + 36(z - 1) &= 0. \\ x + 3y + 9z &= 18. \end{aligned}$$

Problem 2(12 pt) Find all the critical points of $f(x, y) = x^2 + y^2 + x^2y + 4$ **and** classify them (local maximum, local minimum, saddle point).

solution)

$$\begin{aligned} f_x &= 2x + 2xy = 0, \\ f_y &= 2y + x^2 = 0, \quad \implies y = -\frac{1}{2}x^2. \end{aligned}$$

So,

$$\begin{aligned} 2x + 2xy &= 2x + 2x\left(-\frac{1}{2}x^2\right) = 2x - x^3 = 0. \\ x(2 - x^2) &= 0. \\ x &= 0, \quad \sqrt{2}, \quad -\sqrt{2}. \end{aligned}$$

Corresponding y -values are

$$y = 0, \quad -1, \quad -1.$$

Hence, the critical points are

$$(0, 0), \quad (\sqrt{2}, -1), \quad (-\sqrt{2}, -1).$$

Let

$$D(x, y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 + 2y & 2x \\ 2x & 2 \end{vmatrix} = 2(2 + 2y) - 4x^2.$$

Now applying the Second derivative test, we get

(x, y)	$(0, 0)$	$(\sqrt{2}, -1)$	$(-\sqrt{2}, -1)$
$D(x, y)$	4 (+)	-8 (-)	-8 (-)
$f_{xx}(x, y)$	2 (+)	0	0
	local minimum	saddle	saddle