MATH 242 QUIZ 7 NAME (Print your name): Type B

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

Problem 1(8 pt) Find the equation of the tangent plane to the surface $xy^2z^3 = 12$ at $(3, -2, 1)$.

solution) A normal vector to the tangent plane at the point (x, y, z) is

$$\vec{\nabla} F(x, y, z) =< F_x, F_y, F_z >=< y^2z^3, 2xyz^3, 3xy^2z^2 >,$$

where $F(x, y, z) = xy^2z^3$.

So, at $(3, -2, 1)$, we have

$$\vec{\nabla} F(3, -2, 1) =< 4, -12, 36 > .$$

Hence, the equation of the tangent plane to the given surface at $(3, -2, 1)$ is

$$4(x - 3) - 12(y + 2) + 36(z - 1) = 0.$$
$$x - 3y + 9z = 18.$$

Problem 2(12 pt) Find all the critical points of $f(x,y) = x^2 + y^2 + x^2y + 4$ **and** classify them (local maximum, local minimum, saddle point).

solution)

$$f_x = 2x + 2xy = 0,$$
$$f_y = 2y + x^2 = 0, \quad \Longrightarrow y = -\frac{1}{2}x^2.$$

So,

$$2x + 2xy = 2x + 2x(-\frac{1}{2}x^2) = 2x - x^3 = 0.$$
$$x(2 - x^2) = 0.$$
$$x = 0, \quad \sqrt{2}, \quad -\sqrt{2}.$$

Corresponding y-values are

$$y = 0, \quad -1, \quad -1.$$

Hence, the critical points are

$$(0,0), \quad (\sqrt{2}, -1), \quad (-\sqrt{2}, -1).$$

Let

$$D(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2+2y & 2x \\ 2x & 2 \end{vmatrix} = 2(2+2y) - 4x^2.$$

Now applying the Second derivative test, we get

(x,y)	$(0,0)$	$(\sqrt{2},-1)$	$(-\sqrt{2},-1)$
$D(x,y)$	4 (+)	-8 (-)	-8 (-)
$f_{xx}(x,y)$	2 (+)	0	0
	local minimum	**saddle**	**saddle**