MATH 242 QUIZ 8

NAME (Print your name): Type B

SECTION:

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

Problem 1 (20 points) Use Lagrange multipliers to find the maximum and minimum of the function $f(x, y, z) = 6x + 2y + 14z$ with the side condition $x^2 + y^2 + z^2 = 59$.

solution) Let $g(x, y, z) = x^2 + y^2 + z^2$.

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ x^2 + y^2 + z^2 = 59 \end{cases} \Longrightarrow \begin{cases} 6 = 2\lambda x \\ 2 = 2\lambda y \\ 14 = 2\lambda z \\ x^2 + y^2 + z^2 = 59 \end{cases} \Longrightarrow \begin{cases} x = \frac{3}{\lambda},\ y = \frac{1}{\lambda},\ z = \frac{7}{\lambda} \\ \frac{9}{\lambda^2} + \frac{1}{\lambda^2} + \frac{49}{\lambda^2} = 59 \end{cases}$$

$\Longrightarrow \lambda^2 = 1 \Longrightarrow \lambda = \pm 1$.

Now $\lambda = 1$ produces the solution $(3, 1, 7)$ and the value of the function is $18 + 2 + 98 = 118$ while $\lambda = -1$ produces the solution $(-3, -1, -7)$ and the value of the function is $-18 - 2 - 98 = -118$. The first is the absolute max while the second is the absolute min.