MATH 242 QUIZ 9

NAME (Print your name): Type B

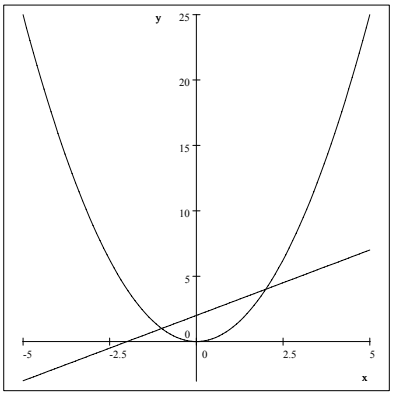
SECTION:

No notes, books, or calculators are allowed. You need to show all your work to get a full credit.

Problem 1(6 pt) Calculate $\int\int_R y\cos(xy)dA$, where $R = [0,1] \times [0,\pi]$.

$$\begin{aligned}\int_0^{\pi}\int_0^1 y\cos(xy)dxdy &= \int_0^{\pi}[\sin(xy)]_{x=0}^1 dy \\ &= \int_0^{\pi}\sin y dy \\ &= [-\cos y]_0^{\pi} \\ &= -\cos(\pi) + \cos(0) \\ &= 2\end{aligned}$$

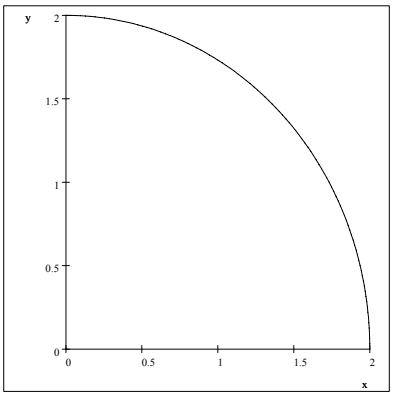
Problem 2(7 pt) Evaluate $\int\int_D dA$, where D is the region bounded by $y = x^2$ and $y = x + 2$.



We first find the points of intersection of $y = x^2$ and $y = x + 2$. Set $x^2 = x + 2$. This gives $x^2 - x - 2 = (x-2)(x+1) = 0$. Hence the points of intersection correspond to $x = 2$ and $x = -1$.

$$\begin{aligned}\int\int_D dA &= \int_{-1}^2\int_{x^2}^{x+2} dydx \\ &= \int_{-1}^2 (x + 2 - x^2)dx \\ &= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^2 \\ &= \frac{3}{2} + 6 - \frac{9}{3} \\ &= \frac{9}{2}\end{aligned}$$

Problem 3(7 pt) Convert $\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2+y^2)dxdy$ to polar coordinates.



If $x = \sqrt{4-y^2}$, then we have $x^2+y^2 = 4$. Hence the region of integration is the part of the disk $x^2+y^2 \leq 4$ that lies in the first quadrant. In polar coordinates we replace x^2+y^2 by r^2 and $dxdy$ by $rdrd\theta$. Hence the given integral is

$$\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{2} r^3 drd\theta$$