

Practice Problems for Exam 2

Problem 1 Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $z = \frac{x}{y} \sin(xy^2)$.

Problem 2 Find $f_x(1, 1)$ and $f_y(1, 1)$ given that $f(x, y) = y - e^{xy^2} + \sqrt{x^2 + 1}$.

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Problem 3 Find the second derivatives of $f(x, y) = \sqrt{16 - 9x^2 - 4y^2}$.

Problem 4 Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{1 + xy}{1 + x^2 + y^2}$.

Problem 5 Using polar coordinates, evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$.

Problem 6 Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

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Problem 7 Find the gradient vector at $(1, 1, 2)$ of $f(x, y, z) = \ln(x^2 + y^2 + z^2)$.

Problem 8 Find the directional derivative of $f(x, y) = 3xy^2 - 4x^3y$ in the direction of $3\vec{i} + 4\vec{j}$.

Problem 9 Find the directional derivative of $f(x, y, z) = x^2y + xy^2 + z^2$ at $(1, 1, 1)$ toward the point $(3, 1, 2)$.

Problem 10 The temperature at (x, y) is given by $T(x, y) = xy - x$. Find a unit vector in the direction in which the temperature increase most rapidly at $(1, 1)$ and find the maximum rate of increase in temperature at $(1, 1)$.

Problem 11 Find the rate of change of $f(x, y, z) = x^2 + y^2 + z^2$ with respect to t along the curve $\vec{r}(t) = 2 \cos 10t \vec{i} + 3 \sin 10t \vec{j} - 20t \vec{k}$.

Problem 12 Use the chain rule to find $\frac{\partial u}{\partial t}$ if $u = \sqrt{x^2 + y^2}$, $x = e^t$, and $y = \sin t$.

Problem 13 Use the chain rule to find $\frac{\partial z}{\partial u}$ if $z = x^3 + xy + y^2$, $x = 2u + v$, and $y = u - 2v$.

Problem 14 Verify that $z = f(x^3 - y^2)$ satisfies the equation $2y \frac{\partial z}{\partial x} + 3x^2 \frac{\partial z}{\partial y} = 0$.

Problem 15 Use the implicit differentiation theorem to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(\frac{1}{2}, -1, 2)$ if $e^{xz} + \ln(yz + 3) - y - 1 - e = 0$.

Problem 16 Find an equation for the tangent plane to the surface $4x^2 + 9y^2 + z = 17$ at the point $(-1, 1, 4)$.

Problem 17 Find a point on the surface $z = 16 - 4x^2 - y^2$ at which the tangent plane is perpendicular to the line $x = 3 + 4t$, $y = 2t$, $z = 2 - t$.

Problem 18 Find the differential df given that $f(x, y, z) = z^4 - 3yz^2 + y \sin z$.

Problem 19 Find the linear approximation of $f(x, y) = \sqrt{x^2 + y^2}$ at $(2, 3)$ and use it to estimate the number $\sqrt{(1.98)^2 + (3)^2}$.

Problem 20 Find the critical point(s) of $f(x, y) = 3xy - 5x^2 - y^2 + 5x - 2y$ and classify the critical point(s).

Problem 21 Find three positive number whose product is 64 and whose sum is as small as possible.

Problem 22 Use the Lagrange multiplier method to find the point on $z^2 = x^2 + y^2$ that is closest to the point $(2, 2, 0)$.