Practice Problems for Exam 3

Problem 1 Calculate the following double integrals.
a) $\int_0^1 \int_{x^2}^x (x^2 + y^2) dy dx$

b) $\int_0^1 \int_0^x e^{x^2} dy dx$

Problem 2 Evaluate $\int_0^1 \int_{2y}^2 \cos(x^2) dx dy$ by interchanging the order of integration.

Problem 3 Use a double integral to find the area bounded by $y = x^2$ and $y = \sqrt{x}$.

Problem 4 Use a double integral to find the volume of the solid in the first octant bounded by the paraboloid $z = x^2 + y^2$ and the planes $z = 0$, $x + y = 1$.

Problem 5 Calculate the average value of $f(x, y) = e^x$ over the region $D: \ 0 \leq x \leq \ln y, \quad 1 \leq y \leq e$.

Problem 6 Use a triple integral to find the volume of the solid in the first octant bounded by the cylinder $x^2 + y^2 = 4$, and the planes $z = y$, $z = 0$.

Problem 7 Calculate $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \frac{1}{\sqrt{x^2+y^2}} dxdy$ by changing to polar coordinates.

Problem 8 Use a double integral in polar coordinates to find the area that is inside $r = 3\sin 3\theta$.

Problem 9 Evaluate $\int_0^1 \int_0^z \int_0^{\sqrt{yz}} x dx dy dz$.

Problem 10 Set up an iterated integral for $\int\int\int_T x dv$, where T is the tetrahedron bounded by $x + y + z = 3$ and the coordinate planes.

Problem 11 Set up a triple integral for the volume of the solid bounded by the cylinder $y^2 = 4x$, and the planes $z = 0$, $z = x$, $x = 4$.

Problem 12 Set up a double integral for the volume of the solid enclosed by $z = x^2 + y^2$, $z = 18 - x^2 - y^2$.

Problem 13 Find the center of mass of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ if the density is proportional to the distance from the yz-plane.

Problem 14 Find the surface ares of the part of the plane $2x+5y+z=10$ that lies in the first octant.

Problem 15 Find the surface of the part of the sphere $x^2+y^2+z^2=4$ that lies within the cylinder $x^2+y^2=2x$ and above the xy-plane.