**Math 285 Quiz 3**

**Problem** Find general solutions of the given differential equations.

1) (15 points) $xy' + 2y = 6x^2\sqrt{y}$.

**solution:** First divide each side by $x$. Then,

$$y' + \frac{2}{x}y = 6x\sqrt{y}, \quad \text{Bernoulli equation with } n = \frac{1}{2}.$$

Let $v = y^{1-\frac{1}{2}} = y^{1/2}$. Then, $y = v^2$ and $\frac{dy}{dx} = 2v\frac{dv}{dx}$.

$$2v\frac{dv}{dx} + \frac{2}{x}v^2 = 6xv.$$

$$\frac{dv}{dx} + \frac{1}{x}v = 3x \quad \text{linear equation.}$$

Integrating factor $\rho(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$.

$$\frac{d}{dx}(x \cdot v) = 3x^2.$$

$$x \cdot v = x^3 + C.$$

$$v = x^2 + Cx^{-1}.$$

$$y^{1/2} = x^2 + Cx^{-1} \quad \text{or} \quad y = (x^2 + Cx^{-1})^2.$$

2) (10 points)$(2xy^3 + e^x)dx + (3x^2y^2 + \sin y)dy = 0$.

**solution:**

Let $M = 2xy^3 + e^x$ and $N = 3x^2y^2 + \sin y$.

$M_y = 6xy^2$, $N_x = 6xy^2$. Hence the equation is exact. We are looking for the function $F(x, y)$ so that $F_x = M$ and $F_y = N$.

$$F(x, y) = \int (2xy^3 + e^x)dx + g(y) = x^2y^3 + e^x + g(y).$$

$$F_y = 3x^2y^2 + g'(y) = 3x^2y^2 + \sin y.$$

$$g'(y) = \sin y \quad \text{hence, } g(y) = -\cos y + C.$$

$$F(x, y) = x^2y^3 + e^x - \cos y = C.$$

**Problem 2** (5 points) Suppose that a population $P(t)$ satisfies

$$\frac{dP}{dt} = P(4 - P).$$

At time $t = 0$, the population is 5. Find the limiting population, i.e, $\lim_{t\to\infty} P(t)$, by using the phase line method.

**solution:** The critical points are where $P(4 - P) = 0$, so $P = 0, 4$. Note that

$$P = \begin{cases} < 0, & \text{if } P < 0 \\ > 0, & \text{if } 0 < P < 4 \\ < 0, & \text{if } P > 4. \end{cases}$$

So, $P$ is decreasing for $P > 4$ and the limiting population is 4.

Note that $P$ is in fact decreasing and concave up for $P > 4$, since $\frac{dP}{dt} = P(4 - P)$ is negative and decreasing if $P > 4$.