

Math 285 Quiz 4

You need to show all your work to get full credit.

Problem 1 a) Use the Wronskian to prove that $f(x) = e^x$ and $g(x) = xe^x$ are linearly independent.

solution)

$$W(f, g) = \begin{vmatrix} e^x & xe^x \\ e^x & (1+x)e^x \end{vmatrix} = e^{2x} \neq 0.$$

b) Show directly that the given functions are linearly dependent.

$$2 \sin^2 x, 4 \cos^2 x, 1.$$

solution)

$$2(2 \sin^2 x) + 4 \cos^2 x + (-4)1 = 0.$$

Hence they are linearly dependent.

Problem 2 Find a solution of the following initial value problem when the two linearly independent solutions y_1, y_2 of the associated homogeneous equation and a particular solution y_p is given.

$$y'' - 2y' - 3y = 6; \quad y(0) = 3, y'(0) = 11;$$

$$y_1 = e^{-x}, \quad y_2 = e^{3x}, \quad y_p = -2.$$

solution)

$$y = y_c + y_p = c_1 e^{-x} + c_2 e^{3x} - 2.$$

$$y' = -c_1 e^{-x} + 3c_2 e^{3x}.$$

Using the initial conditions above, determine c_1 and c_2 .

$$c_1 + c_2 - 2 = 3, \text{ i.e., } c_1 + c_2 = 5,$$

$$-c_1 + 3c_2 = 11.$$

Hence, $c_1 = 1, \quad c_2 = 4.$

The solution is

$$y = e^{-x} + 4e^{3x} - 2.$$