

Practice Problem for Sec 9-7: Polar Coordinates

Solve the following Dirichlet problem for the semicircular disk of radius 3 :  $x^2 + y^2 \leq 9$ ,  $0 \leq \theta \leq \pi$ .

$$\nabla^2 u = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0,$$

$$u_\theta(r, 0) = u_\theta(r, \pi) = 0,$$

$$u(3, \theta) = f(\theta).$$

Assume that the solution  $u$  is bounded, and you may also assume that the eigenvalues and the eigenfunctions for  $x'' + \lambda x = 0$ ,  $x'(0) = x'(L) = 0$  are

$$\lambda_n = \frac{n^2 \pi^2}{L^2}, \quad n = 0, 1, 2, \dots,$$

$$x_n = \cos \frac{n\pi t}{L}, \quad n = 0, 1, 2, \dots$$