

Practice Problems: Chapter 1, Section 2.1 and 2.2:

Problem 1 Determine the order of the given differential equation, and determine if the equation is linear or nonlinear.

(a) $(1 + y)^2 \frac{dy^2}{dx^2} + x \frac{dy}{dx} + y = e^x.$

(b) $\frac{d^3y}{dx^3} + x \frac{dy}{dx} + (\cos^2x)y = x^3.$

Problem 2 Verify that the given function or functions is a solution of the differential equation.

(a) $xy' - y = x^2; y = 3x + x^2.$

(b) $2x^2y'' + 3xy' - y = 0, x > 0; y_1(x) = x^{\frac{1}{2}}, y_2(x) = x^{-1}.$

Problem 3 Decide if the statement is true or false.

(a) The initial value problem $x \frac{dy}{dx} = 2y$, $y(1) = 0$ has a unique solution on some interval containing 1.

(b) The initial value problem $x \frac{dy}{dx} = 2y$, $y(0) = 1$ has infinitely many solutions.

Problem 4

(a) Consider the initial value problem $y' = \sqrt{y}$, $y(0) = 1$. Find the slope of the solution at $x = 0$.

(b) Consider the initial value problem $y' = e^{x+y}$, $y(0) = 1$. Find the slope of the solution at $x = 0$.

Problem 5 Find a general solution of the following differential equations.

(a) $y' + 2y = e^x$.

(b) $(1 - y^2)y' - x^2 = 0$.

$$(c) x^2 y' - xy^2 - 3y^2 = 0$$

$$(d) y' = \frac{x^2 + xy + y^2}{x^2}$$

$$(e) y' + 2xy = \frac{6x}{y^2}$$

$$(f) y' = (x + y - 7)^2$$

Problem 6 Solve the following initial value problems

(a) $xy' + 2y = 4x^2$, $y(1) = 2$.

(b) $(6xy^3 + 2y^4) + (9x^2y^2 + 8xy^3)y' = 0$, $y(1) = 1$.

Problem 7 Mixture Problem: Go over the homework problem 36a) in Section 1.5

Problem 8 Consider a body that moves horizontally through a medium whose resistance is proportional to the square of the velocity v , so that $\frac{dv}{dt} = -kv^2$. Assume that $v(0) = v_0$ and initial position $x(0) = x_0$.

a. Show that $v(t) = \frac{v_0}{1+v_0kt}$.

b. Show that its position function $x(t) = x_0 + \frac{1}{k} \ln(1 + v_0kt)$.

Problem 9 Consider a fish population $P(t)$ given by the following model $\frac{dP}{dt} = P(P - 8)$,
 $P(0) = 5$

(a) Find the critical points.

(b) Use a phase diagram to determine each critical point is stable or unstable.

(c) What can we expect as time goes?

(d) Solve explicitly for $P(t)$.