

Practice Problems: Chapter 3

Problem 1 Find a general solution of the following homogeneous differential equations.

(a) $y'' - 7y' + 12y = 0$

(b) $y^{(3)} - 2y^{(2)} + y^{(1)} = 0$

(c) $y'' - 2y' + 2y = 0$

(d) $(D - 3)^2(D^2 - 6D + 13)y = 0$, where $D = \frac{\partial}{\partial x}$.

Problem 2 Find the appropriate form of a particular solution of the following nonhomogeneous differential equations. Do not evaluate the coefficients.

(a) $y'' - 2y' - 3y = 6$

(b) $y^{(5)} - y^{(3)} = 6$

(c) $y^{(5)} - y^{(3)} = e^x + 2x^2 - 5$

(d) $y'' - 2y' + 2y = e^x \sin x$

(e) $y'' - 7y' + 12y = (x + 2)e^{3x}$

(f) $(D - 3)^2(D^2 - 6D + 13)y = 5e^{3x} \cos 2x + x^2 e^{3x}$

Problem 3 Find a particular solution of $y'' + 9y = \sec 3x$.

Problem 4 (a) Consider $x^2y'' + xy' - y = 0$, $x > 0$. Assuming that the solution is in $y = x^r$ form, find r .

(b) Find a general solution of $x^2y'' + xy' - y = 9x^{-10}$.

Problem 5 Consider the free damped oscillation $x'' + 2x' + 5x = 0$ with initial position $x(0) = 1$ and initial velocity $x'(0) = -1 - 2\sqrt{3}$.

(a) Find its position function.

(b) Find the time lag, envelope curves.

(c) Determine the behavior of $x(t)$ as $t \rightarrow \infty$.

Problem 6 Consider following forced-undamped oscillation $x'' + 9x = \cos \omega t$.

(a) Solve the given differential equation by considering two separate cases $\omega = 3$ and $\omega \neq 3$.

(b) For which value(s) of ω does the resonance occur, i.e., the solution grows and oscillates without bound?

Problem 7 Consider the damped forced oscillation $x'' + 6x' + 9x = 13 \cos 2t$.
(a) Find the transient solution $x_{\text{tr}}(t)$, and determine the behavior as $t \rightarrow \infty$.

(b) Find the steady periodic solution $x_{\text{sp}}(t)$.

Problem 8 Find all the non-negative eigenvalues and the associated eigenfunctions of
 $y'' + \lambda y = 0; y'(0) = 0, y'(\pi) = 0.$