Math 385 B1 Quiz 1

Name (Please Print.): Key

Problem 1 (10 points) Solve the given initial value problem.

$$\frac{dy}{dx} = 2y - 1, \quad y(0) = 1.$$

solution)

$$\frac{dy}{dx} = 2y - 1 : \text{ separable}$$

$$\frac{1}{2y-1}dy = dx$$

$$\int \frac{1}{2y-1}dy = \int dx$$

$$\frac{1}{2}\ln|2y-1| = x + C$$

$$\ln|2y-1| = 2x + 2C$$

$$|2y-1| = e^{2x+2C}$$

$$2y - 1 = De^{2x}$$

$$y = \frac{De^{2x}+1}{2}.$$

$$1 = \frac{D+1}{2} \Rightarrow D = 1.$$

$$\text{Hence, } y = \frac{e^{2x}+1}{2}$$

Problem 2 (5 points) Let $\frac{dy}{dx} = \sqrt{y}$, $y(0) = 0$. Determine if the existence of the solution is guaranteed. If the existence of the solution is guaranteed, determine wheter the uniqueness of the solution is guaranteed.

solution) Note that both $f(x,y) = \sqrt{y}$ *and* $\frac{\partial f}{\partial y} = \frac{1}{2}\frac{1}{\sqrt{y}}$ *are not continuous on any box containing* $(0,0)$. *It is clear that* $\frac{1}{2}\frac{1}{\sqrt{y}}$ *is not continuous at* $(0,0)$, *and hence not continuous near* $(0,0)$. *Since* $\sqrt{y}$ *is defined only for* $y \geq 0$, *we can not find a box containing* $(0,0)$ *on which* $\sqrt{y}$ *is continuous (even defined).*

Hence, the existence and uniqueness theorem does not say anything in this case.

In fact, we can easily find two solutions $y_1 = \frac{1}{4}x^2$ *and* $y_2 \equiv 0$, y_1 *by separation of variable method and* y_2 *by inspection;*

$\frac{dy}{dx} = \sqrt{y} \Rightarrow \frac{1}{\sqrt{y}}dy = dx \Rightarrow \int \frac{1}{\sqrt{y}}dy = \int dx \Rightarrow 2\sqrt{y} = x + C \Rightarrow y = (\frac{x+C}{2})^2$,

$y(0) = 0 \Rightarrow 0 = C.$

Remark: The existence and the uniqueness theorem (roughly speaking, "continuity of $f(x,y)$ near (a,b) guarantees the existence, and the continuity of both f and $\frac{\partial f}{\partial y}$ near (a,b) guarantees the uniqueness") is still good to use for most purposes as in the homework problems.