

Math 385 B1 Quiz 2

Name (Please Print): _____ Key _____

Problem 1 (7 points) Solve the following initial value problem

$$y' + \frac{5}{x}y = 7x, \quad y(2) = 5.$$

solution)

$$y' + \frac{5}{x}y = 7x : \quad \text{first order linear equation}$$

$$\text{Integrating factor } \rho(x) = e^{\int \frac{5}{x} dx} = e^{5 \ln x} = x^5.$$

Multiply through by x^5 to the given equation, we get

$$\frac{d}{dx}(x^5 y) = 7x^6.$$

$$\int \frac{d}{dx}(x^5 y) dx = \int 7x^6 dx.$$

$$x^5 y = x^7 + C.$$

$$y = x^2 + Cx^{-5}.$$

$$y(2) = 5; \quad 4 + \frac{C}{32} = 5.$$

$$C = 32.$$

$$\text{Hence, the solution is } y = x^2 + \frac{32}{x^5}.$$

Problem2 (8 points) Solve $x^2 y' = xy + y^2$ by making a proper substitution.

solution) Rewrite the equation as

$$y' = \frac{y}{x} + \frac{y^2}{x^2} : \quad \text{homogeneous equation}$$

$$\text{Use a substitution } v = \frac{y}{x} \left(\Rightarrow y = vx, \quad \frac{dy}{dx} = x \frac{dv}{dx} + v \right).$$

Then the above equation becomes

$$x \frac{dv}{dx} + v = v + v^2.$$

$$x \frac{dv}{dx} = v^2.$$

$$\frac{1}{v^2} dv = \frac{1}{x} dx.$$

$$\int \frac{1}{v^2} dv = \int \frac{1}{x} dx.$$

$$-\frac{1}{v} = \ln|x| + C.$$

$$v = -\frac{1}{\ln|x| + C}.$$

$$y = vx = -\frac{x}{\ln|x| + C}.$$