Math 385 B1 Quiz 3

Name (Please Print.): Key

Problem 1 Find a general solution of the following equations.

(a) $xy' + 2y = 6x^2\sqrt{y}$.

solution)

$$xy' + 2y = 6x^2\sqrt{y}: \text{ Bernoulli equation with } n = \frac{1}{2}$$

$$y' + \frac{2}{x}y = 6x\sqrt{y}.$$

$$v = y^{1-\frac{1}{2}} = y^{\frac{1}{2}} \Rightarrow y = v^2,\ \frac{dy}{dx} = 2v\frac{dv}{dx}.$$

$$2v\frac{dv}{dx} + \frac{2}{x}v^2 = 6xv.$$

$$\frac{dv}{dx} + \frac{1}{x}v = 3x: \text{ linear first order equation}$$

$$\rho(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x.$$

$$\frac{d}{dx}(xv) = 3x^2.$$

$$xv = \int 3x^2 dx = x^3 + C.$$

$$v = x^2 + \frac{C}{x}.$$

$$y = \left(x^2 + \frac{C}{x}\right)^2.$$

(b) $(2xy^3 + e^x)dx + (3x^2y^2 + \sin y)dy = 0.$

solution) Let $M = 2xy^3 + e^x$ and $N = 3x^2y^2 + \sin y$.
Then, $M_y = 6xy^2,\ N_x = 6xy^2$. Hence, the equation is exact. Thus there exists a $F(x,y) = C$ such that $F_x = M$ and $F_y = N$.

$$F_x = 2xy^3 + e^x,\ F_y = 3x^2y^2 + \sin y.$$

$$F = \int (2xy^3 + e^x)dx + g(y) = x^2y^3 + e^x + g(y)$$

$$\Rightarrow \frac{\partial F}{\partial y} = 3x^2y^2 + \frac{dg}{dy} = 3x^2y^2 + \sin y.$$

$$\frac{dg}{dy} = \sin y.$$

$$g(y) = \int \sin y dy = -\cos y + C.$$

Hence, $F(x,y) = x^2y^3 + e^x - \cos y + C = C_1$.
A general solution is $x^2y^3 + e^x - \cos y = C$.