

Math 385 B1 Quiz 4

Name (Please Print.): Key

Problem 1 Find general solutions of the given differential equations.

(a) (4 points) $y^{(3)} - 2y^{(2)} + y^{(1)} = 0$

solution)

$$r^3 - 2r^2 + r = 0.$$

$$r(r-1)^2 = 0.$$

$$r = 0, 1, 1$$

$$y = c_1e^{0x} + c_2e^x + c_3xe^x = c_1 + c_2e^x + c_3xe^x.$$

(b) (3 points) $y'' + 2y' + 5y = 0$

solution)

$$r^2 + 2r + 5 = 0.$$

$$r = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm 2i.$$

$$y = c_1e^{-x} \cos 2x + c_2e^{-x} \sin 2x (= e^{-x}(c_1 \cos 2x + c_2 \sin 2x)).$$

Problem 2 Let $y'' - 2y' - 3y = 6$. A particular solution y_p of this nonhomogeneous equation is given by $y_p = -2$.

(a) (3 points) Find a general solution of the associated homogeneous equation. solution)

$$r^2 - 2r - 3 = 0.$$

$$(r - 3)(r + 1) = 0.$$

$$r = 3, -1.$$

$$y = c_1 e^{3x} + c_2 e^{-x}.$$

(b) (5 points) Find the solution of the above nonhomogeneous equation with the following initial conditions: $y(0) = 3$, $y'(0) = 11$.

solution) A general solution of the nonhomogeneous equation is

$$y = c_1 e^{3x} + c_2 e^{-x} - 2.$$

$$y' = 3c_1 e^{3x} - c_2 e^{-x}.$$

$$y(0) = 3; c_1 + c_2 - 2 = 3.$$

$$y'(0) = 11; 3c_1 - c_2 = 11.$$

$$c_1 = 4, c_2 = 1.$$

$$y = 4e^{3x} + e^{-x} - 2.$$