Math 385 B1 Quiz 5

Name (Please Print.):______Key_________________________________

Problem (7 points) Consider a free undamped mass-spring system with $m = 3\,\mathrm{kg}$ and $k = 75\,\mathrm{N/m}$. It is set in motion with initial position $x(0) = 1\,\mathrm{m}$ and initial velocity $x'(0) = -5\,\mathrm{m/s}$. Find its position function $x(t)$, and write in the form $C\cos(\omega_0 - \alpha)$ with $\alpha \in (0, 2\pi)$. Also, find the amplitude and the period.

solution)

$$3x'' + 75x = 0.$$

Characteristic equation: $3r^2 + 75 = 0 \Rightarrow r = \pm 5i.$

$$x(t) = c_1 \cos 5t + c_2 \sin 5t.$$

$$x'(t) = -5c_1 \sin 5t + 5c_2 \cos 5t.$$

$$x(0) = 0 : \quad c_1 = 1.$$

$$x'(0) = 0 : \quad 5c_2 = -5 \Rightarrow c_2 = -1.$$

Hence, $x(t) = \cos 5t - \sin 5t$

$$= \sqrt{1+1}\,\cos(5t - \frac{7\pi}{4}) = \sqrt{2}\,\cos(5t - \frac{7\pi}{4}).$$

The amplitude is $\sqrt{2}$, and the period is $\frac{2\pi}{5}$.

Problem 2 Find an appropriate form of a particular solution. Do not evaluate the coefficients.

(a) (3 points) $4y'' + 4y' + y = 3xe^x$

solution)

$$\text{Characteristic equation:} \quad 4r^2 + 4r + 1 = 0$$

$$(2r+1)^2 = 0 \Rightarrow r = -\frac{1}{2}, -\frac{1}{2}.$$

Hence, two linearly independent solutions in y_c are $e^{-\frac{1}{2}x}$ and $x\,e^{-\frac{1}{2}x}$.

Since $f(x) = 3xe^x$, we can set up $y_p = (Ax+B)e^x$.

You do not multiply $(Ax+B)e^x$ by x^s, since there is no duplication.

Hence,

$$y_p = (Ax+B)e^x.$$

(b) (5 points) $y^{(5)} - y^{(3)} = e^x + 2x^2 - 5 + \sin x$

solution)

$$\text{Characteristic equation:} \quad r^5 - r^3 = 0$$

$$r^3(r+1)(r-1) = 0 \Rightarrow r = 0, 0, 0, -1, 1.$$

Hence, linearly independent solutions in y_c are $e^{0x} = 1, x, x^2, e^{-x,}$ and e^x.

Set up a particular solution for each e^x, $2x^2 - 5$, $\sin x$ separately.

e^x : $y_{p_1} = Axe^x$: because of the duplication e^x in y_c, we need to multiply by x.

$2x^2 - 5$: $y_{p2} = x^3(Bx^2 + Cx + D)$: we need to multiply by x^3 to eliminate duplication $1, x, x^2$ in y_c.

$\sin x$: $y_{p2} = E\sin x + F\cos x$: there is no duplication in y_c.

Hence,

$$y_p = Axe^x + x^3(Bx^2 + Cx + D) + E\sin x + F\cos x.$$