Math 385 B1 Quiz 6
Name (Please Print.): Key

Problem 1(8 points) Find a particular solution of $y'' + 4y' + 4y = x^{-2}e^{-2x},\ x > 0$ by using the method of variation of parameters.
(note: $u_1 = -\int \frac{y_2 \cdot f}{W}dx,\ u_2 = \int \frac{y_1 \cdot f}{W}dx$)
solution)

$$r^2 + 4r + 4 = 0.$$
$$(r+2)^2 = 0.$$
$$r = -2, -2$$
$$y_1 = e^{-2x},\ y_2 = xe^{-2x}.$$

$$y_p = u_1y_1 + u_2y_2,$$

where $u_1 = -\int \frac{y_2 \cdot f}{W}dx, u_2 = \int \frac{y_1 \cdot f}{W}dx,$ and $f(x,y) = x^{-2}e^{-2x}.$

$$W(y_1, y_2) = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x} - 2xe^{-2x} \end{vmatrix} = e^{-4x}.$$

$$u_1 = -\int \frac{xe^{-2x}x^{-2}e^{-2x}}{e^{-4x}}dx = -\int \frac{1}{x}dx = -\ln x.$$
$$u_2 = \int \frac{e^{-2x}x^{-2}e^{-2x}}{e^{-4x}}dx = \int \frac{1}{x^2}dx = -\frac{1}{x}.$$

Hence,

$$y_p = (-\ln x)(e^{-2x}) + \left(-\frac{1}{x}\right)(xe^{-2x}) = -e^{-2x}\ln x - e^{-2x}.$$

Note that since e^{-2x} is a solution of associated homogeneous equation, you can simply take $y_p = -e^{-2x}\ln x$.

Problem 2 (7 points) Find the steady perioduc solution of $x'' + \frac{3}{2}x' + x = 18\cos 2t$, and write the solution in $C\cos(\varpi t - \alpha)$ form with $C > 0$ and $0 < \alpha < 2\pi$.
solution)

$$\begin{aligned}
x_{\text{sp}} &= A\cos 2t + B\sin 2t. \\
x_{\text{sp}}' &= 2B\cos 2t - 2A\sin 2t. \\
x_{\text{sp}}'' &= -4A\cos 2t - 4B\sin 2t. \\
x_{\text{sp}}'' + \frac{3}{2}x_{\text{sp}}' + x_{\text{sp}} &= (-4A + 3B + A)\cos 2t + (-4B - 3A + B)\sin 2t \\
&= 18\cos 2t. \\
-3A + 3B &= 18, \\
-3B - 3A &= 0. \\
A &= -3, \text{ and } B = 3.
\end{aligned}$$

Hence,

$$\begin{aligned}
x_{\text{sp}} &= -3\cos 2t + 3\sin 2t \\
&= \sqrt{9+9}\cos(2t - \frac{3\pi}{4}) \\
&= 3\sqrt{2}\cos(2t - \frac{3\pi}{4}).
\end{aligned}$$

Note that α is on the Quad II, and $\arctan 1 = \frac{\pi}{4}$.