Math 385 B1 Quiz 9

Name (Please Print.):____

Problem 1 (15 points) By separating variables and making use of the fact below, find the solution of

$$u_t = 2u_{xx}, \quad 0 < x < 2,$$
$$u(0,t) = u(2,t) = 0.$$
$$u(x,0) = x.$$

Leave the coefficient in integral form. You do not hve to evaluate it.

Useful facts: For the boundary value problem, $X'' + \lambda X = 0, \quad X(0) = X(L) = 0$, the only nonzero solution X_n (eigenfunctions), together with the values λ_n(eigenvalues) which produce them, are

$$X_n = \sin\frac{n\pi x}{L}, \quad \lambda_n = \frac{n^2\pi^2}{L^2}, \quad n = 1,2,3,\ldots$$

solution) Set $u(x,t) = X(x)T(t)$. Then,

$$XT' = 2X''T$$
$$\frac{X''}{X} = \frac{T'}{2T} = -\lambda.$$

So,

$$X'' + \lambda X = 0; X(0) = X(2) = 0$$

Hence, $X_n = \sin\frac{n\pi x}{2}$ for each eigenvalue $\lambda_n = \dfrac{n^2\pi^2}{4}$, $n = 1,2,\ldots$. Now, solve for T for each $\lambda_n = \dfrac{n^2\pi^2}{4}$, $n = 1,2,\ldots$.

$$\frac{T'}{2T} = -\frac{n^2\pi^2}{4}.$$
$$\frac{dT}{T} = -\frac{n^2\pi^2}{2}dt.$$
$$\ln|T| = -\frac{n^2\pi^2}{2}t + C.$$
$$T_n = Ce^{-\frac{n^2\pi^2 t}{2}}, n = 1,2,\ldots.$$

Thus,

$$u(x,y) = \sum_{n=1}^{\infty} c_n X_n T_n = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{2} e^{-\frac{n^2\pi^2 t}{2}}.$$

Since $u(x,0) = 0$, we have $\sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{2} = x$. Hence,

$$c_n = \int_0^2 x\sin\frac{n\pi x}{2}dx.$$