Math 285 Section G8, Quiz Number 2

October 25, 2000

Name: ANSWER KEY

1. (3 points) Some of the differential equations below are autonomous, some are not. Similarly, some are separable, some are not. In each case, circle or underline the questions for which the answer is "true".

$y'(t) = y^2 - 2y + 1$	$\boxed{\textbf{is autonomous}?}$	$\boxed{\textbf{is separable}?}$
$y'(t) = y + t$	**is autonomous**?	**is separable**?
$y'(t) = -t/y$	**is autonomous**?	$\boxed{\textbf{is separable}?}$

2. (2 points) Use separation of variables to solve the differential equation

$$y'(t) = y^2(t), \qquad y(0) = 1.$$

Rewrite this as $\frac{dy}{dt} = y^2$, *so*

$$\frac{1}{y^2}\,dy = 1\,dt$$

Integrate both sides:

$$\int \frac{1}{y^2}\,dy = \int 1\,dt$$

$$-1/y = t + C \tag{1}$$

Plug in $t = 0$ *into equation* (1) *to find* C*:*

$$-1/y(0) = -1 = C$$

Plugging $C = -1$ *into* (1), *we get*

$$-1/y = t - 1$$

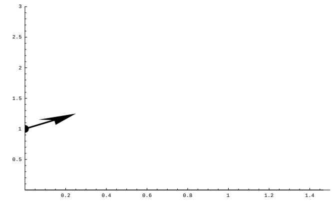
And finally, solving for $y(t)$, *we get*

$$y(t) = \frac{1}{1-t}$$

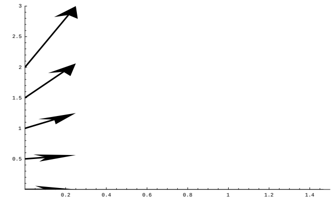
NOTE: Several students gave the answer $y(t) = 1$ for this problem. Even though this agrees with the initial condition $y(0) = 1$, with a little thought it should be obvious that $y(t) = 1$ cannot be right. The reason is that $y(t) =$ constant means $y'(t) = 0$. But we know $y'(0) = y^2(0) = 1$, by combining the differential equation with our initial condition. More about this on the following page . . . .

The plots below were produced with *Mathematica*, but it should not be hard to give a rough sketch by hand.

Just looking at $t = 0$, it is not hard to work out a one-point "flow plot" that starts with our initial condition of $y(0) = 1$. As explained on the previous page, we also need to compute $F(t, y) = y^2$ at this point. This is enough to see that $y(t)$ is not constant, must be headed uphill.



With a bit more work, we can get a flow plot for different possible starting conditions given by different possible choices of $y(0)$:



Finally, since this differential equation is autonomous (doesn't depend on t), we can get a more complete flow plot just by copying our "$t = 0$ flowplot above, plugging in the same arrows for various values of t: